Path and Path Deviation Equations in Kaluza-Klein Type Theories: A Brief Introduction

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Abstract

Path and path deviation equations for charged, spinning and spinning charged objects in different versions of Kaluza-Klein (KK) theory using a modified Bazanski Lagrangian have been derived. The significance of motion in five dimensions, especially for a charged spinning object, has been examined. We have also extended the modified Bazanski approach to derive the path and path deviation equations of a test particle in a version of non-symmetric K-K theory.

1 Introduction

In an attempt to unify gravity and electromagnetism Kaluza(1921) introduced a fifth dimension to describe electromagnetism. Klein(1926) added a stringent cylindrical condition , which keeps the extra dimension compact[1]. Following the scheme of compactification many theories have developed KK ideas and extended the process of compactification to include higher dimensions as a way to unify many fields[2]. However, Wesson et al. [3] have considered unification of geometry with matter by dropping the cylindrical condition, and introducing non-compact theories of higher dimensions based geometrically on the Campbell -Magaard theorem. This approach has been emerged into two classes of non-compact theories: brane theories [4] and space-time -matter theories [5].

From this perspective, path and path deviation equations play a vital part to interpret the behavior of any particle describing any of the above mentioned theories. These equations offer a way to test the new physics coming from the introduction of extra dimensions[6]. The behavior of test particles and extended objects could be used for examining additional phenomena embedded in higher dimensions. Accordingly, we present a study of path and path deviation equations for charged, spinning and spinning charged objects using different theories of KK. The path and path deviation equations could also be used to detect the cosmological variation of

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spin, and to study the evolution of the angular momentum of galaxies, pulsars and high energy primordial objects [7] using a gyroscopic motion in 5-dimensions. KK theories have been extended to include different types of non-symmetric theory of gravity. One such trial has been done by Kalinowski to unify gravity and gauge fields using a multidimensional manifold in the Jordan-Thirry manner[8].

The aim of the present work is to extend the Bazanski approach[9] into 5D in order to derive some versions of path and path deviation equations in multidimensional space for different objects such as charged, spinning and spinning charged particles, with taking the status of extra dimension as either compact or non-compact. Moreover, we are going to apply the Bazanski approach to derive the path and path deviation equations for a test particle moving in non-symmetric theories of gravitation in 5D .

The paper is organized into the following steps:
(1) Describing path equations and their corresponding path deviation equations in 4D.
(2) Extending these equations into 5D.
(3) Comparing and contrasting each equation in compact spaces with the corresponding equations for non-compact spaces.

2 Motion in 4D

2.1 Path & Path Deviation Equations in Riemannian Geometry

Geodesic and geodesic deviation equations can be obtained simultaneously by applying the action principle on the Bazanski Lagrangian [9]:

$$L = g_{\alpha\beta} U^{\alpha} \frac{D\Psi^{\beta}}{Ds}, \tag{1}$$

where $\frac{D}{Ds}$ is the covariant derivative. This can be done if one takes the variation with respect to the deviation vector Ψ^{ρ} in order to derive the geodesic equation:

$$\frac{dU^{\alpha}}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^{\mu} U^{\nu} = 0, \tag{2}$$

where $\left\{ {\alpha \atop \mu\nu} \right\}$ is the Christoffel symbol. If one takes the variation with respect to the unit tangent vector U^{ρ}, one derives the geodesic deviation equation:

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\beta\gamma\delta} U^{\beta} U^{\gamma} \Psi^{\delta}. \tag{3}$$

where $R^{\alpha}_{\beta\gamma\delta}$ is the Riemann- Christoffel curvature tensor. It is worth mentioning that the Bazanski approach has been successfully applied in geometries different from the Riemannian [10],[11]. Now, Lagrangian (1) can be amended to describe path and path deviation equations of charged, spinning and spinning charged particles if we introduce the following Lagrangian:

$$L = g_{\alpha\beta} U^{\alpha} \frac{D\Psi^{\beta}}{Ds} + f_{\beta} \Psi^{\beta} \tag{4}$$

such that

$$f_\beta = a_1 F_{\alpha\beta} U^\beta + a_2 R_{\alpha\beta\gamma\delta} S^{\gamma\delta} U^\alpha,$$

where a_1 and a_2 are parameters that may take the values $\frac{e}{m}$ and $\frac{1}{2m}$ respectively,$F^\mu_{.\nu}$ is an Electromagnetic tensor and $S^{\gamma\delta}$ is the spin tensor . These parameters have to be adjusted with their counterparts in the original Lorentz force equation [12], the Papapetrou equation [13] and the Dixon equation [14].

Applying the Bazanski approach for obtaining path and path deviation equations on Lagrangian(4) we get:
(i)The Lorentz charged equation (for $a_1 = \frac{e}{m}$ and $a_2 = 0$)

$$\frac{dU^\alpha}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^\mu U^\nu = \frac{e}{m} F^\mu_{.\nu} U^\nu, \tag{5}$$

and the charged deviation equation [15]:

$$\frac{D^2\Psi^\alpha}{Ds^2} = R^\alpha_{.\mu\nu\rho} U^\mu U^\nu \Psi^\rho + \frac{e}{m}(F^\alpha_{.\nu}\frac{D\Psi^\nu}{Ds} + F^\alpha_{.\nu;\rho} U^\nu \Psi^\rho). \tag{6}$$

(ii)The Papapetrou equation for spinning objects (for $a_1 = 0$ and $a_2 = \frac{1}{m}$)

$$\frac{dU^\alpha}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^\mu U^\nu = \frac{1}{2m} R^\alpha_{.\mu\nu\rho} S^{\nu\rho} U^\mu. \tag{7}$$

The spinning deviation equation becomes [16]:

$$\frac{D^2\Psi^\alpha}{Ds^2} = R^\alpha_{.\mu\nu\rho} U^\mu U^\nu \Psi^\rho + \frac{1}{2m}(R^\alpha_{.\mu\nu\rho} S^{\nu\rho}\frac{D\Psi^\mu}{Ds} + R^\alpha_{\mu\nu\lambda} S^{\nu\lambda}_{.;\rho} U^\mu \Psi^\rho + R^\alpha_{\mu\nu\lambda;\rho} S^{\nu\lambda} U^\mu \Psi^\rho) \tag{8}$$

(iii)The Dixon Equation for spinning charged objects(for $a_1 = \frac{e}{m}$ and $a_2 = \frac{1}{m}$) [14]:

$$\frac{dU^\alpha}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^\mu U^\nu = \frac{e}{m} F^\mu_{.\nu} U^\nu + \frac{1}{2m} R^\alpha_{.\mu\nu\rho} S^{\nu\rho} U^\mu, \tag{9}$$

and its spinning charged deviation equation becomes:

$$\frac{D^2\Psi^\alpha}{Ds^2} = R^\alpha_{.\mu\nu\rho} U^\mu U^\nu \Psi^\rho + \frac{e}{m}(F^\alpha_{.\nu}\frac{D\Psi^\nu}{Ds} + F^\alpha_{.\nu;\rho} U^\nu \Psi^\rho) + \frac{1}{2m} R^\alpha_{.\mu\nu\rho} U^\mu U^\nu \Psi^\rho$$
$$+ \frac{1}{2m}(R^\alpha_{.\mu\nu\rho} S^{\nu\rho}\frac{D\Psi^\mu}{Ds} + R^\alpha_{\mu\nu\lambda} S^{\nu\lambda}_{.;\rho} U^\mu \Psi^\rho + R^\alpha_{\mu\nu\lambda;\rho} S^{\nu\lambda} U^\mu \Psi^\rho). \tag{10}$$

Papapetrou [17] has derived an equation describing a spinning object which is able to precess:

$$\frac{D}{Ds}(mU^\alpha + U_\rho \frac{DS^{\alpha\rho}}{Ds}) = \frac{1}{2} R^\alpha_{.\mu\nu\rho} S^{\nu\rho} U^\mu. \tag{11}$$

Using the Bazanski approach, we can suggest the following Lagrangian:

$$L = g_{\alpha\beta}(mU^\alpha + U_\rho \frac{DS^{\alpha\rho}}{Ds})\frac{D\Psi^\beta}{Ds} + R_{\alpha\beta\gamma\delta} S^{\gamma\delta} U^\beta \Psi^\alpha, \tag{12}$$

which can be used to derive equation (11) and to obtain its corresponding deviation equation in the following way:

$$\frac{D^2\Psi^\alpha}{Ds^2} = R^\alpha_{.\mu\nu\rho} U^\mu (mU^\nu + U_\beta \frac{DS^{\nu\beta}}{Ds})\Psi^\rho + g^{\alpha\sigma} g_{\nu\lambda}(mU^\lambda + U_\beta \frac{DS^{\lambda\beta}}{Ds})_{;\sigma} \frac{D\Psi^\nu}{Ds}$$
$$+ R^\alpha_{.\mu\nu\rho} S^{\nu\rho}\frac{D\Psi^\mu}{Ds} + R^\alpha_{\mu\nu\lambda} S^{\nu\lambda}_{.;\rho} U^\mu \Psi^\rho + R^\alpha_{\mu\nu\lambda;\rho} S^{\nu\lambda} U^\mu \Psi^\rho. \tag{13}$$

2.2 Path & Path Deviation Equations in Non-Symmetric Geometries

Path equations in one of the versions of non-symmetric geometries e.g. Legaré and Moffat have been derived from following Lagrangian [18]

$$L = g_{(\mu\nu)} U^{\mu} U^{\nu} + \lambda \hat{A}_{\nu} U^{\nu}, \qquad (14)$$

by taking the variation with respect to U^{σ} to give

$$\frac{dU^{\alpha}}{ds} + \left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\} U^{\mu} U^{\nu} = \lambda g^{(\alpha\mu)} f_{[\mu\nu]} U^{\nu} \qquad (15)$$

where $g_{(\mu\nu)}$ is the symmetric part of the gravitational potential tensor,λ is a parameter and, $f_{[\mu\nu]} = \hat{A}_{\mu,\nu} - \hat{A}_{\nu,\mu}$ is a skew symmetric tensor related to the Yukawa force.

Applying the Bazanski approach, we can derive (15) from the following Lagrangian:

$$L = g_{(\alpha\beta)} U^{\alpha} \frac{D\Psi^{\beta}}{Ds} + \lambda f_{\nu} \Psi^{\nu}. \qquad (16)$$

Using the same approach, we can show its corresponding deviation equation to be:

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\mu\nu\rho} U^{\mu} U^{\nu} \Psi^{\rho} + \lambda (f_{\alpha.\nu} \frac{D\Psi^{\nu}}{Ds} + f^{\alpha}_{.\nu;\rho} U^{\nu} \Psi^{\rho}). \qquad (17)$$

But, if we consider the following Lagrangian:

$$L = \mathbf{g}_{\mu\nu} U^{\mu} \frac{D\Psi^{\nu}}{D\tau} + \lambda f_{\nu} \Psi^{\nu}, \qquad (18)$$

where $\mathbf{g}_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]}$, and follow the Bazanski approach to get the path and path deviation equations related to this type of geometry by taking the variation with respect to Ψ^{σ} and U^{σ} respectively, we can obtain:

$$\frac{dU^{\alpha}}{ds} + \left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\} U^{\mu} U^{\nu} = \lambda \mathbf{g}^{\alpha\mu} f_{[\mu\nu]} U^{\nu} + \mathbf{g}^{\alpha\sigma} g_{[\nu\sigma];\rho} U^{\nu} U^{\rho}, \qquad (19)$$

and the path deviation equation becomes:

$$\frac{D^2\Psi^{\alpha}}{Ds^2} = R^{\alpha}_{.\mu\nu\rho} U^{\mu} U^{\nu} \Psi^{\rho} + 2\mathbf{g}^{\sigma\alpha} (g_{[\nu[\sigma];\rho]}) \frac{D\Psi^{\nu}}{Ds} U^{\rho} + \lambda (f^{\alpha}_{.\nu} \frac{D\Psi^{\nu}}{Ds} + f^{\alpha}_{.\nu;\rho} U^{\nu} \Psi^{\rho}). \qquad (20)$$

It is clear that the difference between (15) and (19) is related to absence of the spin of the source in (15). Thus, from (19) it is possible to find an interaction between the spin of the source and the skew field [19]. Kalonowski [20] has extended Moffat's version [21] which is described in Einstin-Cartan geometry, to establish a relation between the mass and fermion current curves space-time while the spin of the source is twisting it.

Moreover, Wanas and Kahil have extended the Bazanski approach, applying it in Einstein non-symmetric geometries[22] to reach the conclusion that paths in these geometries are naturally quantized (in the Planck sense of quantization)[10]. This type of natural quantization of paths exists in absolute parallelism geometries as well [11].

3 Motion in 5D

The problem of motion in higher dimensions is an intriguing problem. The significance of motion in higher dimensions may yield some indications with regards to the principles that should be followed when describing motion in 4-dimensions, i.e an equation which governs the motion in 4-dimensions [5]. In the present work, we will examine the effect of non gravitational forces on the current motion, i.e. should this motion be absorbed into the extra dimension or remain unchanged from the usual equation of motion in 4-dimensional space apart from increasing the dimensions?

3.1 The Bazanski Approach in 5-Dimensions

In an attempt to derive path and path deviation equations in 5-dimensions, we extend the Bazanski Lagrangian to 5D:

$$L = g_{AB} U^A \frac{D\Psi^B}{DS} \tag{21}$$

where ($A = 1, 2, 3, 4, 5$). By taking the variation with respect to the deviation vector Ψ^C and the tangent vector U^C, we obtain the geodesic and geodesic deviation equations respectively,

$$\frac{dU^C}{dS} + \left\{ {C \atop AB} \right\} U^A U^B = 0, \tag{22}$$

and

$$\frac{D^2\Psi^C}{DS^2} = R^C_{.ABD} \Psi^D U^A U^B \tag{23}$$

3.2 Compact Spaces

The process to unify electromagnetism (gauge fields) and gravity depends on extra component(s) of the metric using the cylinder condition [1]. Some authors believe that compact dimensions can be tested, for example, by examining the rate of energy released as a result of gravitational waves from binary pulsars [23].

In our study, we derive the same geodesic and geodesic deviation equations given by Kerner et al. [15] using the Bazanski approach:

$$\frac{dU^\mu}{dS} + \left\{ {\mu \atop \nu\lambda} \right\} U^\nu U^\lambda + (\frac{dx^5}{dS} + A_\nu \frac{dx^\nu}{dS}) F^{.\mu}_{.\lambda} U^\lambda = 0, \tag{24}$$

$$\frac{d}{dS}(\frac{dx^5}{dS} + A_\mu \frac{dx^\mu}{dS}) = 0. \tag{25}$$

where $Q \equiv \frac{dx^5}{dS} + A_\mu \frac{dx^\mu}{dS}$ is constant along the 5D geodesics i.e.

$$\frac{q}{m} = \frac{dx^5}{dS} + A_\mu \frac{dx^\mu}{dS}.$$

Consequently, (24) becomes:

$$\frac{dU^\mu}{dS} + \left\{ {\mu \atop \nu\lambda} \right\} U^\nu U^\lambda + \frac{q}{m} F^{.\mu}_{\lambda} U^\mu = 0, \tag{26}$$

with $ds^2 = (1 - Q^2)dS^2$ and its corresponding path deviation equation becomes:

$$\frac{D^2\Psi^\mu}{DS^2} = R^\mu_{.\rho\nu\lambda}U^\rho U^\nu \Psi^\lambda + \frac{q}{m}(F^\mu_{.\nu;\rho}U^\nu \Psi^\rho + F^\mu_{.\nu}\frac{D\Psi^\nu}{DS}) + F^\mu_{.\lambda}U^\lambda(\frac{d}{dS}(A_\lambda\Psi^\lambda + \Psi^5) + F_{\nu\rho}U^\nu\Psi^\rho) \quad (27)$$

and

$$\frac{d}{dS}((\Psi^5 + A_\lambda\Psi^\lambda) + F_{\lambda\rho}U^\rho\Psi^\lambda) = 0 \quad (28)$$

A charged particle whose behavior is described by the Lorentz equation in 4D behaves as a test particle moving on a geodesic in 5D. This result is obtained from the usual Basanski method in 5D rather than its modified method in 4D.

3.3 Non-Compact Spaces

In an attempt to unify geometry and matter, Wesson and his collaborators[3] have assumed that $g_{AB,5} \neq 0$, which is applied in the brane world models[4] and space-time-matter theories [5]. The idea of non-compact spaces is based upon the Campbell-Maagard theorem [24]. Using this approach, Wesson [25] has found that:
(1)Massive particles travelling on a time-like geodesic in 4-dim can be regarded as traveling on a null-geodesic in 5D. This is obvious as an implication of the behavior of wave-like particles in a double slits experiment.
(2) Massive particles travelling on any path may exhibit changes their rest mass because there is a direct contact with the fifth force. In this case, the path equation will be a generalization of the problem of moving particles having variable mass in classical mechanics.

It is well known that the path equation of a charged object described in non-compact space [26] is given by

$$\frac{dU^\alpha}{dS} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} U^\mu U^\nu = nF^\alpha_\mu U^\nu U^\nu + \epsilon n^2 \frac{\Phi^{;\alpha}}{\Phi^3} - A^\alpha \frac{dn}{dS} - g^{\alpha\lambda}\frac{dx^5}{dS}(nA_{\lambda,5} + g_{\lambda\mu,5}\frac{dx^\mu}{dS}) \quad (29)$$

and

$$\frac{d}{dS}\epsilon\Phi^2(\frac{dx^5}{dS} + A_\mu\frac{dx^\mu}{dS}) = 0 \quad (30)$$

where $n = \epsilon\Phi^2(\frac{dx^5}{dS} + A_\mu U^\mu)$, Φ is a scalar potential, and $\epsilon = \pm 1$ depending on whether the extra dimension is space-like or time-like respectively. This leads to $\frac{q}{m} = \epsilon\Phi^2(\frac{dx^5}{dS} + A_\mu U^\mu)$ in which its scalar field affects the ratio of charge to mass.

However, the above equation has two main defects: it is not gauge invariant, and the additional extra force from an extra dimension is parallel to the four vector velocity i.e. $f_\mu U^\mu \neq 0$.

Ponce de Leon [26] has dealt with these two defects by using various types of transformations in order to make (29) and (30) like the geodesic equation in its usual form:

$$\frac{d^2\xi^A}{dS^2} + \begin{Bmatrix} A \\ BC \end{Bmatrix} \frac{d\xi^B}{dS}\frac{d\xi^C}{dS} = 0, \quad (31)$$

where ξ^A is the projected 5D velocity. This allow us to introduce its corresponding Bazanski Lagrangian:

$$L = g_{AB}\frac{d\xi^A}{dS}\frac{D\Psi^B}{DS} \quad (32)$$

which gives its geodesic deviation equation as:

$$\frac{D^2\Psi^A}{DS^2} = 0 \tag{33}$$

3.4 Path & Path Deviation Equations of Non-Symmetric Geometries in 5D

We now consider the following Lagrangian:

$$L = \mathbf{g}_{AB} U^A \frac{D\Psi^B}{DS} + \lambda f_C \Psi^C, \tag{34}$$

Applying the Bazanski approach to derive the path and path deviation equations by taking the variation with respect to Ψ^D and U^D respectively, we obtain:

$$\frac{dU^A}{dS} + \left\{ {A \atop BC} \right\} U^B U^C = \lambda \mathbf{g}^{AD} f_{[DC]} U^C + \mathbf{g}^{AD} g_{[CD];M} U^C U^M, \tag{35}$$

and

$$\frac{D^2\Psi^A}{DS^2} = R^A_{.BCD} U^B U^C \Psi^D + 2\mathbf{g}^{DA} (g_{[A[D];M]}) \frac{D\Psi^C}{DS} U^M + \lambda \mathbf{g}^{DA} (f_{A.C} \frac{D\Psi^C}{Ds} + f_{AC;M} U^C \Psi^M). \tag{36}$$

In one version of K-K Non-symmetric theory of gravity, Kalnowski [8] has summarized the role of the extra dimension in the following matter: mass and fermion current curve the four dimensions, the spin of the source and the electromagnetic potential twist the fifth dimension.

4 Rotation in 5D

The concept of rotation in higher dimensions is related to obtaining the governing equation of the current spinning object [27]. For a spinning gyroscope it is well known that the fifth equation is testing the rate of precession [28]. Some authors believe that the study of two nearby free-falling gyroscopes could be used to examine the question of the existence of gravitational waves[16].

We may apply the Bazanski approach on the following Lagrangian:

$$L = g_{AB} U^A \frac{D\Psi^B}{DS} + \frac{1}{2m} R_{ABCD} S^{CD} U^B \Psi^A \tag{37}$$

to derive the path equation of a spinning object in 5D:

$$\frac{dU^C}{dS} + \left\{ {C \atop AB} \right\} U^A U^B = \frac{1}{2m} R^C_{.ABD} S^{BD} U^A \tag{38}$$

The above equation describes spinning objects in compact spaces which satisfy the cylinder condition i.e $g_{AB,5} = 0$. This is identical to the Dixon equation if we project it into four dimensions. The fifth coordinate will contribute the electromagnetic tensor, which has already appeared in the Dixon equation.
Also, the original Papapetrou equation in 5D will be as follows:

$$\frac{D}{DS}(mU^A + U_E \frac{DS^{AE}}{DS}) = \frac{1}{2} R^A_{.BCD} S^{CD} U^B \tag{39}$$

and its corresponding deviation equation will take the following form:

$$\frac{D^2\Psi^A}{DS^2} = R^A_{.BCD} U^B (mU^C + U_E \frac{DS^{CE}}{DS})\Psi^D + g^{AC} g_{BE} (mU^E + U_O \frac{DS^{EO}}{DS})_{;C} \frac{D\Psi^B}{DS}$$
$$+ R^A_{.BCD} S^{CD} \frac{D\Psi^B}{DS} + R^A_{BCE} S^{CE}_{.;D} U^B \Psi^D + R^A_{BCE;D} S^{CE} U^B \Psi^D. \quad (40)$$

These equations could be used to study the behavior of spinning charged objects that exhibit precession e.g. neutron stars, compact objects..etc.

In non-compact spaces with $R_{ABCD} = 0$, it is found that a spinning particle is moving on a geodesic in 5D rather than the Papapetrou equation [29]. This leads us to suggest that in non-compact spaces, satisfying the Campbell-Magaard theorem, spinning particles and spinning charged particles as well as test particles are moving along geodesics in 5D. But if we consider the original Papapetrou equation in 5D, we can find out that it is different from the usual geodesic equation i.e.

$$\frac{D}{Ds}(mV^A + V_B \frac{DS^{AB}}{DS}) = 0. \quad (41)$$

On the contrary, its corresponding deviation equation is identical to (33).

5 Discussion and Conclusion

The Lagrangian required to derive the path and path deviation equations in higher dimensions becomes the conventional Bazanski Lagrangian if the extra fields are described in the fifth (higher) dimension. Path equations of different particles in 4D can be considered as the projection of the geodesic equation in 5D on 4D. But if the extra effect, non gravitational force, is not totally absorbed in the higher dimensional equations then the Bazanski Lagrangian should be amended like their counterparts in 4-dimension, which is seen in the 5D Papapertrou's equation(38).

It has been shown in this paper that the effect of the compactness of the extra dimension(s) can be clearly perceived on moving objects. Also, we find that the effect of precession may distinguish between tops and test particles moving in a background whose 5D curvature has vanished.

In our study we have shown that the apparent Papapetrou equation in 5D is merely the projection of the Dixon equation in 4D. But if the space does not include electromagnetism as an extra dimension, the Papapetrou equation in 4D remains the same in 5D unless the extra dimension space is not compact satisfying the Campbell-Magaard theorem. Also, as can be seen from (25) and (30) the path equations in compact and non-compact spaces display a contradictory aspect: the ratio of charge to mass is constant in the case of compact space, while it is variable depending on the scalar field in case of non-compact spaces, which can be explored due to the effect of the cylinder condition on higher dimensions.

It was shown that in 4D, the Bazanski Lagrangian can be modified to describe path equations for charged, spinning and spinning charged particles, as well as for spinning objects with precession and their corresponding path deviation equation. In an attempt to find the path and path deviation equations of the above mentioned particles in 5D, we have found that the Bazanski Lagrangian could remain unmodified if the non gravitational force could be absorbed into the higher dimension. Otherwise, the Bazanski Lagrangian must be amended.

In our study, we have also found that in non-compact spaces, spinning objects and the non precessing ones are not following the same trajectory, although their path deviation equations are the same.

We have applied the Bazanski approach to determine path and path deviation equations of one version of the Einstein non-symmetric theories of gravity in 5D. This work could be extended to study the effect of compactness on the path equations in our future work.

In addition to our study, we would like to point out some trends based on extending more than one time-like dimension. Recently, Chen [30] has shown that it is possible to interpret the double slit experiment using one than one time-like dimension. However, using Wesson's approach [25], which is based on non-compact extra dimension, one could interpret the same effect. Also Chen [30] has obtained an equation of paths with different spins; these types of equations have also been obtained by Wanas [31] and applied by Wanas et al. to interpret the discrepancy of the COW-experiment [32] and to provide a consistent temporal model of SN1987A using parameterized absolute parallelism geometry in 4D [33]. So, introducing more than one time-like dimension to solve some problems in nature is a tempting suggestion that needs further discussion.

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