Dark Matter: The Problem of Motion

Article *in* Gravitation and Cosmology · January 2018

DOI: 10.1134/S020228931903006X

Magd Elias Kahil
Modern Sciences and Arts University

Dark Matter: The Problem of Motion

January 26, 2018

Magd E. Kahil [1 2 3]

Abstract

Equations of non-geodesic and non-geodesic deviations for different particles are obtained, using a specific type of classes of the Bazanski Lagrangian. Such type of paths has been found to describe the problem of variable mass in the presence of Riemannian geometry. This may give rise to detect the effect of dark matter which reveals the mystery of motion of celestial objects that are not responding neither to Newtonian nor Einsteinian gravity. An important link between non-geodesic equations and the dipolar particle or fluids has been introduced to apply the concept of geometization of physics. This concept has been already extended to represent the hydrodynamic equations in a geometric way. Such an approach, demands to seek for an appropriate theory of gravity able to describe different regions, eligible for detecting dark matter. Using different versions of bi-metric theory of gravity, to examine their associate non-geodesic paths. Due to implementing the geometrization concept, the stability problem of non-geodesic equations are essential to be studied for detecting the behavior of those objects in the presence of dark matter .

1 Introduction

The quest of flat rotational curves for spiral galaxies cannot be explained neither classical nor general relativistic gravity, such a violation can be regarded to the existence of dark matter. In our Galaxy, several meticulous observations have confirmed that, the rotational velocities are ranging between $200 \sim 300 km/s$, based on considering it as a function of the distance r from the SgrA*. Taking into account that those clouds are moving in circular orbits with velocity $v_g(r)$, leading to the relationship between the mass profile $M(r)$ and rotational velocities in the following way [1],

$$M(r) = r v_g^2(r)/G, \tag{1}$$

[1]Modern Sciences and Arts University , Giza , Egypt
[2]Nile University,Giza, Egypt
e.mail: mkahil@msa.eun.eg
magdelias.kahil@gmail.com
[3]Egyptian Relativity Group. Cairo, Egypt

Consequently, the problem now is clarified on studying this stringent motion and revealing its violation in different characteristics which gives some elusiveness to obtain a unique explanation for this type of discrepancy between theory and observation.

Accordingly, one may find out that dark matter (DM) can be detected as a mass excess quantity, expressed in non-geodesic equation due to the projection of the fifth component of the geodesic of non-compact higher dimensional theory of gravity on its 4-dimensions [1], or considering it as a result of the motion of dipolar particles rather than test particles on these clouds. In more detail, dark matter can be presented, by a set of equations of linear momentum and internal momentum :the evolution equation. In this approach, we suggest a corresponding Lagrangian, similar to the spinning Lagrangian [4] , having the flavor of the Bazanski-like Lagrangian [3],[4]. Following this trend, we can utilize this Lagrangian for examining the stability of a system affected by dark matter, by solving the deviation equations of the path and evolution respectively.

Nevertheless, the notation of DM has not fixed only in the region of halos, but there are some traces that may confirm its existence at different regions in the universe or inside the core of the Galaxy. It is attributed to determine the precise value of the cosmic microwave spectrum, providing a reliable scenario of the abundances of elementary particles of large scale structures from Big Bang nucleosynthesis [5]. From this perspective, it is really essential to find a consistent theory of gravity able to reveal these variant regions, having some features of strong field theory of gravitation.

Thus, we focus on different versions of bi-metric theories of gravity. These theories have the ability to express dark matter as a twin matter appeared in the Halo , by means of a ghost free bi-gravity theory or a mass-excess term appeared in a fluid circumventing the core of the galaxy, as in case of SgrA*, using a specific bi-metric theory having geodesic mapping to be acting as gauge transformation [6].

Thus, it is well known that DM is a matter of elusiveness. Some authors consider it as weak interacting massive particles, others may view it not only a stream of perfect fluids, but also dipolar fluids [7] to be matched with descriptions of MOND - weak gravitational fields and BIMOND- strong gravitational fields; which can be detected on the halos and nearby the core of the galaxy within the accretion disk [8].

Yet, there is a rival approach, should be considered , rejecting the existence of dark matter and dark energy [DE] revealed as expressed in terms of MOND [9]and BIMOND paradigms [10]. This type of illustration led Blanchet et al to modify the appearance of dark matter in their model to behave from a particle content into a fluid-like behavior.[2]

From this perspective, it is vital in our study to derive the candidate equations of motion showing the mass excess term is due to the existence of dark matter. Nevertheless, a vital question should be addressed: What is dark matter?
In our present work, we present three rival explanations for its contents:
Firstly: The projection of a higher dimension spatial dimension on the 4-dim manifold? [1]
Secondly: Motion of dipolar particles /fluids as claimed in spiral galaxies? [2]
Thirdly: The existence of a scalar field associated with the Galaxy's gravitational field? [10]

[4]See Apendix A

Moreover, there are several remedies to modify Newtonian and/or Einsteinan Gravity by changing their gravitational potentials to become like $\phi = -GM[1+\alpha \exp(-r/r_0)]/(1+\alpha)$, with $\alpha = -0.9$ and $r_0 \approx 30kpc$, ini order to explain the behavior of the flat rotational curves for spiral galaxies.[1]

These varieties of explanations may let us argue about its dominance, Recently, some authors have indicated that DM occupies 26% of invisible matter of the Universe, while 73% of Universe's composition is made of DE due to expansion with acceleration [11][Iroiro]. Yet, the link between DM and DE is becoming one of vital problems, it has been found that MOND paradigm has a link between DM and DE which in terms of ΛCDM. Such an argument may lead to a crucial speculation that the existence of DM is not localized in the halos of the galaxy, but it could be visualized differently in accretion disk or through very minute discrepancies of perihelion motion for some planets [12].

In our work we are not localizing dark matter to be detected only on the halos of spiral galaxies , but also in the core of the galaxy. Accordingly, the need to search for an eligible theory of gravity able to describe all these ones. One of the candidates is examining the motion of bi-gravity type theories. These theories have adopted two times of matter: ordinary matter and twin matter. The interaction between twin matter and ordinary matter may give rise to the significance of the existence of dark matter.

Consequently, the problem of realizing the notation of dark matter remains unsolved unless, one presents its associate gravitation field theory. In our work, we are going to apply different versions bi-metric theories [13] as candidates to examine its existence, the first is the Hassan-Rosen [14] version for galactic region; while, the second, a special case of conformal type, is presented by Verzub's description of bi-metric [6], which is successfully related to strong gravitational field as in the core of the galaxy is based on the strength of the gravitational field, nearby the core can be expressed as mass excess appeared in hydrostatic equation of the flow of fluid on the accretion [15].

Accordingly, in the present work , we are going to deal with expressing, the behavior of dark matter in terms of non-geodesic equations, these equations are derived using the Lagrangian formalism using the Bazanski-like Lagrangian. This type of equation may give rise to geometrize all trajectories associated with the appearance of dark matter. In other words, the appropriate path equations as described in the Riemannian geometry to represent dipolar particles or fluids of the halos; and the corresponding path equations that represents the hydrostatic stream of fluids in of accretion disk.due to solving the non-geodesic deviation equation, we can give rise to examine stability conditions, which means an indication of remaining DM effect on each observed regions. Thus, we are able to examine the equivalence of non-geodesic trajectories with each of the following equations, dipolar moment and dipolar fluids and hydrostatic stream of motion as described in General Relativity is explained in section 2 . We extend the previous equations to be expressed in different versions of bi-metric theories of gravity as shown in sec 3. Finally, it turns out that, the problem of detecting the existence of DM is expressed in terms studying the behavior of the stream of fluids in different gravitational fields.

This may raise the necessity to examine the stability of these systems for being affected by dark matter. This can be seen, by solving the different corresponding deviation equation for examining the stability condition, using an independent method of coordinate transformation [16] which is be described in sec 4.

2 Dark Matter: Equations of Motion in GR

2.1 Dark Matter: Non-Geodesic Equations

Paths that follow non-geodesic trajectories, and their corresponding deviation equations are obtained using the Euler-Lagrange equation of the following Lagrangian:

$$L \stackrel{def.}{=} m(s) g_{\mu\nu} U^{\mu} \frac{D\Psi^{\nu}}{Ds} + m(s)_{,\rho} \Psi^{\rho} \tag{2}$$

such that:

$$\frac{d\partial L}{ds \partial \dot{\Psi}^{\alpha}} - \frac{\partial L}{\partial \Psi^{\alpha}} = 0 \tag{3}$$

One obtains,

$$\frac{dU^{\alpha}}{ds} + \Gamma^{\alpha}_{\beta\delta} U^{\beta} U^{\delta} = \frac{m(s)_{,\beta}}{m(s)} (g^{\alpha\beta} - U^{\alpha} U^{\beta}) \tag{4}$$

and using the commutation relation (A.4) and the condition (A.5) we obtain its corresponding deviation equation;

$$\frac{D^2 \Psi^{\mu}}{Ds^2} = R^{\mu}_{\nu\rho\sigma} U^{\nu} U^{\rho} \Psi^{\sigma} + (\frac{m(s)_{,\beta}}{m(s)} (g^{\alpha\beta} - U^{\alpha} U^{\beta}))_{;\rho} \Psi^{\rho} \tag{5}$$

Assuming that the effective mass $m(s) \sim exp(g(\psi)\psi)$, ,which may contribute to describe the behavior of the parallel force , $f_{||} = \nabla[g(\psi)\psi]$ as shown on the right hand side of equation.

It is well known that this force is responsible for mass variation of paths in the presence of the Riemaniann geometry. Yet, it has been found [17] by taking σ as another parameter , such that $s \sim \sigma$ to obtain the following relation

$$\frac{1}{m} \frac{dm}{d\sigma} \equiv \frac{\sqrt{\Lambda/2}}{6} \tag{6}$$

in which to be expressed as,

$$\frac{1}{m} \frac{dm}{d\sigma} \approx 2a_0/c \tag{7}$$

where a_0 is a constant of acceleration, $a_0 \sim 2 \times 10^{-10} m/sec^2$, as known of the MOND and c is the speed of light. Thus, we can find that the non-geodesic equation can be related to MOND [9] in the following way:

$$\frac{d\hat{U}^{\alpha}}{d\sigma} + \Gamma^{\alpha}_{\beta\delta} \hat{U}^{\beta} \hat{U}^{\delta} = 2\frac{a_0}{c} \hat{U}_{\beta} (g^{\alpha\beta} - \hat{U}^{\alpha} \hat{U}^{\beta}) \tag{8}$$

where, $\hat{U}^{\alpha} = \frac{dx^{\alpha}}{d\sigma}$ and its corresponding deviation equation becomes

$$\frac{D^2 \hat{\Psi}^{\mu}}{D\sigma^2} = R^{\mu}_{\nu\rho\sigma} \hat{U}^{\nu} \hat{U}^{\rho} \hat{\Psi}^{\sigma} + 2\frac{a_0}{c} (\hat{U}_{\beta} (g^{\alpha\beta} - \hat{U}^{\alpha} \hat{U}^{\beta}))_{;\rho} \hat{\Psi}^{\rho} \tag{9}$$

such that $\hat{\Psi}^{\alpha}$ is its associated deviation vector.

2.2 Dark Matter: An Extra-dimensional Effect

The non-geodesic equations are expressed as , the four components of a geodesic equations for a test particle [1] in a non-compact space-time $g_{AB,5} \neq 0$ following Wesson's approach of space-time-matter[17]. Thus, the characteristics of dark matter can be appeared within solving the geodesic equation in 5-dim., provided that

$$\frac{dS}{ds} = \sqrt{(1 + \epsilon\hat{\Phi}^2(U^5)^2)}$$

such that $\hat{\Phi}$ is a scalar function, and $\epsilon = \pm 1$.
Thus, it can possible to suggest the following Lagrangian:

$$L = g_{AB} U^A \frac{D\Psi^A}{DS} \tag{10}$$

By taking the variation with respect to Ψ^C and U^C respectively, one can find
(i) Equation of Geodesic:

$$\frac{DU^C}{DS} = 0, \tag{11}$$

(2) Equation of Geodesic Deviation:

$$\frac{D^2\Psi^C}{DS^2} = R^C_{BDE} U^B U^D \Psi^E \tag{12}$$

With taking into account that the force appeared on its right hand side is expressed within the component of the fifth dimension of a 5-dim manifold. Accordingly, equation (4) may be expressed as

$$\frac{d^2x^\mu}{dS} + \Gamma^\mu_{AB} \frac{x^A}{dS} \frac{x^B}{dS} = 0$$

while,the fifth component of (11) plays as a vital to affect the behavior of dark matter particles as present in the other components .

$$\frac{d^2x^5}{dS} + \Gamma^5_{AB} \frac{x^A}{dS} \frac{x^B}{dS} = 0$$

2.3 Dark Matter: Equations of Motion Dipolar Moment Particles in The Halo

The rotational curves of the galaxy can be expressed by the presence of dipolar dark Matter particles [18]. From this perspective, it has been found that these particles are described of two equations, one may describe the (passive) linear momentum and the other describes the microscopic (active) momentum, in terms of the evolution equation. The Lagrangian formalism for motion of the dipole moment in a gravitational field is analogous to the its counterpart the motion of spinning with precession [19][5].

[5] see Appendix A

Thus, we suggest the following Lagrangian:

$$L \stackrel{def.}{=} g_{\alpha\beta} P^{\alpha} \frac{D\Psi^{\beta}_{(1)}}{Ds} + \Omega_{\alpha} \frac{D\Psi^{\beta}_{(2)}}{Ds} + f_{\alpha} \Psi^{\alpha}_{(1)} + \hat{f}_{\alpha} \Psi^{\alpha}_{(2)} \tag{13}$$

such that $\Psi^{\mu}_{(1)}$ is the non-geodesic deviation from the world line and $\Psi^{\mu}_{(2)}$ is the evolution deviation due to dipole moment, with taking into account that the projector tensor is responsible for raising and lowering indices for the Dipole moment vector i.e.

$$\Omega^{\mu} = h^{\mu\nu} \Omega_{\nu}$$

By taking the variation with respect to Φ^{μ}_{1} and Φ^{μ}_{2} separately we obtain the following set of equation of motion and evolution respectively:

$$\frac{DP^{\mu}}{Ds} = f^{\mu} \tag{14}$$

and

$$\frac{D\Omega^{\mu}}{Ds} = \hat{f}^{\mu} \tag{15}$$

such that

$$f^{\mu} = 2mF^{\mu}$$

and

$$\hat{f}^{\mu} = R^{\mu}_{\nu\rho\sigma} \Pi^{\sigma} U^{\rho} U^{\nu}$$

in which

$$\Pi^{\mu} = h^{\mu\nu} \pi_{\nu}$$

Using (A.4) and (A.5) as in the previous section, we obtain their corresponding geodesic deviation equations

$$\frac{D^2\Psi^{\mu}_{(1)}}{DS^2} = R^{\mu}_{\nu\rho\sigma} P^{\nu} U^{\rho} \Psi^{\sigma}_{(1)} + f^{\mu}_{;\rho} \Psi^{\rho}_{(1)}, \tag{16}$$

And,

$$\frac{D^2\Psi^{\mu}_{(2)}}{DS^2} = R^{\mu}_{\nu\rho\sigma} \Pi^{\nu} U^{\rho} \Psi^{\sigma}_{(2)} + \hat{f}^{\mu}_{;\rho} \Psi^{\rho}_{(2)} \tag{17}$$

Equations (16), (17) are essentially vital to solve the problem of stability for different celestial objects in various gravitational fields.

2.4 Equations of Motion of Dipolar Fluid in The Halos

Recently, it has been the involvement of cosmological constant is vital to identify the mystery of dark matter. This led Blanchet et al to revisit the description of of dipolar dark matter from particle contents into fluid-like description [2] , this can be found by imposing the effect of polarization vector as a candidate to examine the interaction of dark energy on the system .

From this perspective, Blanchet and Le Tiec [8] have postulated that the dynamics of the dipolar fluid in a prescribed gravitational field $g_{\mu\nu}$ is derived from an action of the type found

$$S = \int d^4x\sqrt{-g}L[J^\mu, \xi^\mu \dot{\xi}, g_{\mu\nu}] \tag{18}$$

Provided that the density current J^μ and the polarization vector Π^μ are new quantities added in dipolar fluids: such that: $J^\mu = \rho U^\mu$, and $\Pi^\mu = \rho \xi^\mu$ Applying the least action principle on (18) to obtain their corresponding set of path equations

$$\frac{DJ^\mu}{Ds} = 0 \tag{19}$$

and

$$\frac{D\Omega}{Ds} = \frac{1}{\sigma}\nabla^\mu(W - \hat{\Pi}\hat{W}) - R^\mu_{\rho\nu\lambda}u^\rho \xi^\nu K^\lambda$$

where, The above set of equations can be obtained using its associated Bazanski-Like Lagrangian, if we take the variation with respect to $\Psi^\mu_{(1)}$ and $\Psi^\mu_{(2)}$ simultaneously

$$L = g_{\mu\nu}J^\mu \frac{D\Psi^\nu_{(1)}}{Ds} + \Omega_\mu \frac{D\Psi^\nu_{(2)}}{Ds} + \bar{f}_\mu \Psi^\mu_2. \tag{20}$$

Also, using the commutation rule (A.4) and the condition (A.5) we obtain their corresponding path deviation and evolution deviation equations respectively

$$\frac{D^2\Psi^\mu_{(1)}}{Ds^2} = R^\mu_{\nu\rho\sigma}J^\nu U^\rho \Psi^\sigma_1, \tag{21}$$

and

$$\frac{D^2\Psi^\mu_{(2)}}{Ds^2} = R^\mu_{\nu\rho\sigma}\Omega^\nu U^\rho \Psi^\sigma_2 + \bar{f}^\mu_{;\rho}\Psi^\rho_2. \tag{22}$$

2.5 Equations of Motion of Fluids in The Accretion Disk

The role of non-geodesic equations are acting to describe geometrically the behavior of dark matter particles in the accretion disk, as existed as a collision-less fluid. In this section, we are going to focus on its contribution to mass of the accretion desk and consequently, the accretion process is less efficient than that expected from dissipative fluid ; dark matter gives a significant contribution to the mass of the accretion desk producing an important inflow as in our Galaxy, e.g. a mass growth scaling as $M_{bh} = const.t^{9/16}$ [8].

Thus, we can find out that the equivalence between non-geodesic motions and hydrodynamics flows appears in following two sets of equations

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\beta\delta}U^\beta U^\delta = f^\alpha \tag{23}$$

wheref^α is described as non-gravitational force, in which its vanishing turns the equation into a geodesic, which becomes

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\beta\delta}U^\beta U^\delta = \frac{1}{E+P}h^{\alpha\beta}P_{,\beta} \tag{24}$$

where $h^{\alpha\beta}$ is the projection tensor defined as:

$$h^{\mu\nu} = g^{\mu\nu} - U^{\mu}U^{\nu}, \tag{25}$$

If equation (23) satisfies the first law of thermodynamics

$$P_{,\beta} = \rho c^2 (\frac{E+P}{\rho c^2})_{,\beta} \tag{26}$$

then it becomes,

$$\frac{dU^{\alpha}}{ds} + \Gamma^{\alpha}_{\beta\delta} U^{\beta} U^{\delta} = \frac{(\frac{E+P}{\rho c^2})_{,\beta}}{(\frac{E+P}{\rho c^2})} h^{\alpha\beta} \tag{27}$$

Thus, we find out that from ...

$$\frac{1}{E + P/\rho} \frac{dE + P/\rho}{d\sigma} \approx 2a_0/c \tag{28}$$

Such a result is inevitable to ensure that the stream of hydrodymamics equations may be expressed with respect to the MOND constant, for arbitrary parameters defining the motion.

Meanwhile, in case of isobaric pressure, the equation of stream becomes conditionally equivalent to geodesic. Thus, the appearance of the extra term on the right hand side of equation (...) inspire many authors to interrelate it with the problem of dark matter as an excess of mass due to the Lagrangian suggested by Kahil and Harko (2009) [1]:

From the above equations, we can find that the excess of mass for a test particle is equivalent to the hydrodynamic equation of motion for a perfect fluid satisfying the first law of thermodynamics. Such an analogy is required to describe the behavior of cluster of fluid circumventing the AGN it has detected that annihilation of dark matter particles in terms of increase γ ray density in the accretion disk)[21]
Accordingly, we can obtain the hydrodynamic flow of accretion desk by applying the Euler-Lagrange equation on (1) with taking into account that

$$m(s) \stackrel{def.}{=} \frac{(P+E)}{\rho c^2} \tag{29}$$

3 Dark Matter : Equations of Motion in Bimetric Theories

In this section, owing to implementing the concept of geometrization of physics, it is essential to express the motion of non-geodeisc equations and their corresponding deviation equation to regulate the behavior of as expressed in particle content or fluid-like in the presence of different bi-metric gravitational fields, able to explain its presence at different scales.

3.1 Non-Geodesic Trajectories for Bi-gravity

Hossenfelder [22] has introduced an alternative version of bi-metric theory, having two different metrics **g** and **h** of Lorentzian signature on a manifold **M** defining the tangential space TM and co-tangential space T*M respectively. These can be obtained in terms of two types of matter and twin matter; existing individually , each of them has its own field equations as defined within Riemannian geometry. It is well known that implementing bi-gravity theory, without cosmological constants, will be vital to describe motion of dipolar objects in the halos [23]; while the conformal type may be able to describe dark matter as mass excess quantities found in as in accretion disk circumventing the center of the Galaxy, as described by strong gravitational fields.
Meanwhile, theories of bi-metric theories, have one metric combining the two metrics, with cosmological constant, describing variable speed of light to replace the effect dark energy in big bang scenario [24].
From the previous versions of bi-metric theories [25], we are going to present a generalized form which can be present different types of path and path deviation which can be explained for any bi-metric theory which has two different metrics and curvatures as defined by Riemannian geometry [26] . Their Corresponding Lagrangian can be expressed in the following way [27]

$$L \stackrel{def.}{=} m_g g_{\mu\nu} \Psi_{;\nu} U^{\mu} U^{\nu} + m_f f_{\mu\nu} \Phi_{|\nu} V^{\mu} V^{\nu} + (\frac{m_g(s)_{,\beta}}{m_g(s)} (g^{\alpha\beta} - U^{\alpha} U^{\beta}))_{;\rho} \Psi^{\rho} + (\frac{m_f(\tau)_{,\beta}}{m_f(\tau)} (g^{\alpha\beta} - V^{\alpha} V^{\beta}))_{;\rho} \Psi^{\rho}, \tag{30}$$

Thus, considering

(1) $\frac{d\tau}{ds} = 0$.
This will give to two separate sets of path equations owing to each parameter by applying the following Bazanski-like Lagrangian:

$$\frac{DU^{\alpha}}{DS} = \frac{m_{(g)}(s)_{,\beta}}{m_{(g)}(s)} (g^{\alpha\beta} - U^{\alpha} U^{\beta}), \tag{31}$$

and

$$\frac{DV^{\alpha}}{D\tau} = \frac{m_{(f)}(\tau)_{,\beta}}{m_{(f)(\tau)}} (f^{\alpha\beta} - V^{\alpha} V^{\beta}) \tag{32}$$

and their corresponding path deviation equations:

$$\frac{D^2 \Psi^{\alpha}}{DS^2} = R^{\alpha}_{\beta\gamma\delta} U^{\gamma} U^{\beta} \Psi^{\delta} + (\frac{m_{(g)}(s)_{,\beta}}{m_{(g)}} (g^{\alpha\beta} - U^{\alpha} U^{\beta}))_{\rho} \Psi^{\rho}, \tag{33}$$

And,

$$\frac{D^2 \Phi^{\alpha}}{D\tau^2} = S^{\alpha}_{\beta\gamma\delta} V^{\gamma} V^{\beta} \Phi^{\delta} + (\frac{m_{(f)}(\tau)_{,\beta}}{m_{(f)}} (f^{\alpha\beta} - V^{\alpha} V^{\beta}))_{;\rho} \Phi^{\rho}, \tag{34}$$

(2) $\frac{d\tau}{dS} \neq 0$ [26]

The two metrics can be related to each other by means of a quasi-metric one [28].

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu} + \alpha_g(g_{\mu\nu} - U_\mu U_\nu) + \alpha_f(f_{\mu\nu} - V_\mu V_\nu). \quad (35)$$

Such an assumption may give rise to define its related Lagrangian of Bazanski's flavor to describe the geodesic and geodesic deviation equation due to this version of bi-gravity theory.

$$L \stackrel{def.}{=} \tilde{g}_{\alpha\beta} U^\alpha \frac{\tilde{D}\Psi^\beta}{\tilde{D}S}, \quad (36)$$

$$\tilde{\Gamma}^\alpha_{\beta\sigma} = \frac{1}{2}\tilde{g}^{\alpha\delta}(\tilde{g}_{\sigma\delta,\beta} + \tilde{g}_{\delta\beta,\sigma} - \tilde{g}_{\beta\sigma,\delta})$$

and its corresponding Lagrangian:

$$L = \tilde{m(s)}\tilde{g}_{\mu\nu}\tilde{U}^\mu(\frac{d\tilde{\Psi}^\nu}{d\tilde{S}} + \tilde{\Gamma}^\nu_{\rho\delta}\tilde{\Psi}^\rho\tilde{U}^\delta) + \tilde{f}_\mu\Psi^\mu \quad (37)$$

Thus, equation of its path equation can be obtained by taking the variation respect to Ψ^μ to obtain:

$$\frac{d\tilde{U}^\alpha}{d\tilde{S}} + \tilde{\Gamma}^\alpha_{\beta\delta}\tilde{U}^\beta\tilde{U}^\delta = \frac{\tilde{m(S)}_{,\beta}}{\tilde{m(S)}}(\tilde{g}^{\alpha\beta} - \tilde{U}^\alpha\tilde{U}^\beta) \quad (38)$$

and using the commutation relation (A.4) and the condition (A.5), we obtain its corresponding deviation equation;

$$\frac{D^2\Psi^\mu}{\tilde{DS}^2} = \tilde{R}^\mu_{\nu\rho\sigma}\tilde{U}^\nu\tilde{U}^\rho\tilde{\Psi}^\sigma + (\frac{\tilde{m(\tilde{S})}_{,\beta}}{m(\tilde{S})}(\tilde{g}^{\alpha\beta} - \tilde{U}^\alpha\tilde{U}^\beta))_{;\rho}\tilde{\Psi}^\rho \quad (39)$$

where

$$\tilde{R}^\alpha_{.\mu\nu\rho} = \tilde{\Gamma}^\alpha_{\mu\rho,\nu} - \tilde{\Gamma}^\alpha_{\mu\nu,\rho} + \tilde{\Gamma}^\sigma_{\mu\rho}\tilde{\Gamma}^\alpha_{\sigma\rho} - \tilde{\Gamma}^\sigma_{\mu\rho}\tilde{\Gamma}^\alpha_{\sigma\rho}$$

3.2 Equations of Dipolar Moment in Bi-gravity Theory

Equation of motion of diplar moment in the presence of bi-metric theory as a candidate to represent DM as interaction between ordinary and twin matter as described by bi-gravity ghost-free theory.

Accordingly, we suggest the following Lagrangian;

$$L \stackrel{def.def.}{=} g_{\alpha\beta}P^\alpha\frac{D\Psi^\beta_{(1)}}{Ds} + \Omega_\alpha\frac{D\Psi^\beta_{(2)}}{Ds} + f_\alpha\Psi^\alpha_{(1)} + \hat{f}_\alpha\Psi^\alpha_{(2)} + f_{\alpha\beta}Q^\alpha\frac{D\Phi^\beta_{(1)}}{D\tau} + \Delta_\alpha\frac{D\Phi^\beta_{(2)}}{D\tau} + k_\alpha\Phi^\alpha_{(1)} + \hat{k}_\alpha\Psi^\alpha_{(2)} \quad (40)$$

where, Q twin matter momentum vector Δ twin matter dipole moment vector ,j Twin non-gravitational force to momentum , Twin non-gravitational force of dipole moment Thus, taking the variation with respect to Ψ_1, Ψ_2 , Φ_1 and Φ_2 we obtain: the dipolar momentum of ordinary matter, the evolution equation of ordinary matter, the equation of twin dipolar momentum and the equation of twin evolution dipolar moment

$$\frac{DP^\mu}{Ds} = f^\mu \quad (41)$$

Evolution equation of dipolar moment

$$\frac{D\Omega^{\mu}}{Ds} = \hat{f}^{\mu} \tag{42}$$

While, for twin matter Equation of dipolar twin matter

$$\frac{DQ^{\mu}}{D\tau} = k^{\mu} \tag{43}$$

and Evolution equation of twin dipolar moment

$$\frac{D\Delta^{\mu}}{D\tau} = \hat{k}^{\mu} \tag{44}$$

Also, in order to obtain their corresponding deviation equations following the same procedures in for both metrics g and f independently, we get after some manipulations the following set of deviation equations for ordinary matter and twin matter as follows; for the ordinary matter.

$$\frac{D^2\Psi^{\mu}_{(1)}}{DS^2} = R^{\mu}_{\nu\rho\sigma}P^{\nu}U^{\rho}\Psi^{\sigma}_{(1)} + f^{\mu}_{;\rho}\Psi^{\rho}_{(1)}, \tag{45}$$

and

$$\frac{D^2\Psi^{\mu}_{(2)}}{DS^2} = R^{\mu}_{\nu\rho\sigma}\Pi^{\nu}U^{\rho}\Psi^{\sigma}_{(2)} + \hat{f}^{\mu}_{;\rho}\Psi^{\rho}_{(2)} \tag{46}$$

and for the twin matter

$$\frac{D^2\Phi^{\mu}_{(1)}}{D\tau^2} = S^{\mu}_{\nu\rho\sigma}Q^{\nu}V^{\rho}\Phi^{\sigma}_{(1)} + k^{\mu}_{;\rho}\Phi^{\rho}_{(1)}, \tag{47}$$

and

$$\frac{D^2\Phi^{\mu}_{(2)}}{D\tau^2} = S^{\mu}_{\nu\rho\sigma}\hat{\Pi}^{\nu}V^{\rho}\Phi^{\sigma}_{(2)} + \hat{k}^{\mu}_{;\rho}\Phi^{\rho}_{(2)} \tag{48}$$

where, $S^{\alpha}_{\beta\gamma\delta}$, V^{α}, $\hat{\Pi}^{\alpha}$ their associated curvature, for vector velocity, Polarization vector for particles defined as twin matter respectively.

3.3 Dipolar Fluid in Bi-gravity Theory

Extending the previous section to be expressed in bi-gravity ghost free theory to describe both ordinary fluid and twin fluid as described in bi-gravity theory, we suggest the following Lagrangian;

$$L \stackrel{def.}{=} g_{\mu\nu}J^{\mu}\frac{D\Psi^{\nu}_{(1)}}{Ds} + \Omega_{\mu}\frac{D\Psi^{\nu}_{(2)}}{Ds} + f_{\mu\nu}\hat{J}^{\mu}\frac{D\Psi^{\nu}_{(1)}}{D\tau} + \Delta_{\mu}\frac{D\Psi^{\nu}_{(2)}}{D\tau} \tag{49}$$

taking the variation with respect to Ψ_1, Ψ_2 , Φ_1 and Φ_2 we obtain

for Ordinary fluid

$$\frac{DJ^{\mu}}{Ds} = 0$$

and

$$\frac{D\Omega}{Ds} = \frac{1}{\sigma}\nabla^{\mu}(W - \hat{\Pi}\hat{W}) - R^{\mu}_{\rho\nu\lambda}u^{\rho}\xi^{\nu}K^{\lambda}$$

and for twin fluid

$$\frac{D\hat{J}^{\mu}}{D\tau} = 0$$

and

$$\frac{D\Delta}{D\tau} = \frac{1}{\sigma}\nabla^{\mu}(\tilde{W} - \tilde{\Pi}\tilde{W}) - S^{\mu}_{\rho\nu\lambda}V^{\rho}\tilde{\xi}^{\nu}\tilde{K}^{\lambda}$$

3.4 Non-Geodesic Euations in AGN: Bimetric theory

The bi-metric version of equation (4) can be obtained by obtaining the Euler-lagrange equation on the following Lagrangian

$$\tilde{L} = \tilde{g}_{\alpha\beta}\tilde{U}^{\alpha}\frac{D\tilde{\Psi}^{\beta}}{D\tilde{s}} \tag{50}$$

To obtain the corresponding path equation

$$\frac{d\tilde{U}^{\alpha}}{d\tilde{s}} + \tilde{\Gamma}^{\alpha}_{\beta\delta}\tilde{U}^{\beta}\tilde{U}^{\delta} = \frac{\tilde{m}(s)_{,\beta}}{\tilde{m}(\tilde{s})}(\tilde{g}^{\alpha\beta} - \tilde{U}^{\alpha}\tilde{U}^{\beta}) \tag{51}$$

and using the commutation relation (A.4) and the condition (A.5) we obtain its corresponding deviation equation;

$$\frac{D^2\tilde{\Psi}^{\mu}}{D\tilde{s}^2} = \tilde{R}^{\mu}_{\nu\rho\sigma}\tilde{U}^{\nu}\tilde{U}^{\rho}\tilde{\Psi}^{\sigma} + \left(\frac{\tilde{m}(s)_{,\beta}}{\tilde{m(\tilde{s})}}(\tilde{g}^{\alpha\beta} - \tilde{U}^{\alpha}\tilde{U}^{\beta})\right)_{;\rho}\tilde{\Psi}^{\rho} \tag{52}$$

4 Dark Matter: Problem of Stability

4.1 The Relationship between Stability and Geodesic Deviation

The importance of solving deviation equations that are obtained with its path equation for an object whether is counted to be a test particle or not is inevitably used for testing the stability of the system by a perturbation to the geodesic equations [29]is dependent of the style of coordinate system .

This approach has been applied previously in examining the stability of some cosmological models using two geometric structures [30]. Recently, Then above approach has been modified by introducing an approach introduced by [18]] , based on stability condition as a result of by obtaining the scalar value of the deviation vector which gives rise to become independent of any coordinate system which works for examining the stability problem for any planetary system, being a covariant coordinate independent which can be explained in the following way. More over this condition has been implemented to test the stability of stellar systems orbiting strong gravitational fields [32]

This approach is shown in the following way:
Let $\Psi^{\alpha}(S)$ is obtained from the solutions of the deviation equation in a given interval [a,b] in which $\Psi^{\alpha}(S)$ behave monotonically. These quantities can become sensors for measuring the stability of the system are

$$q \stackrel{def.}{=} \lim_{s \to b} \sqrt{\Psi^{\alpha}\Psi_{\alpha}}. \tag{53}$$

If

$$q \to \infty$$

then the system is unstable, otherwise it is always stable.

where C^{α} are constants and $f(S)$ is a function known from the metric. If $f(S) \to \infty$, the system becomes unstable otherwise it is stable. For geodesic or non-geodesic deviation equations

$$q \stackrel{def.}{=} \lim_{s \to b} \sqrt{\Psi^{\alpha}\Psi_{\alpha}}. \tag{54}$$

If $q \to \infty$ then the system is unstable, otherwise it is always stable.

Now, in case of dipolar moment equation there will be another condition for the evolution equation, we suggest the above condition be extended to include deviation tensor $\Phi^{\mu\nu}_{(2)}$ as

$$q_{(2)} \stackrel{def.}{=} \lim_{s \to b} \sqrt{\Psi^{\alpha}\Psi_{(2)\alpha}}. \tag{55}$$

Thus, for such a member in stellar/planetary system is stable, if and only if the magnitude of the scalar value of both spin deviation vectors ${\Phi_2}^{\alpha}$ and evolution deviation tensors $\Phi_2^{\alpha\beta}$ to be real numbers respectively. i.e. either $q_1 \to \infty$ or $q_1 \to \infty$ the assigned member is unstable. Accordingly, a strong stability condition must be admitted if both $q_{(1)}$ and $q_{(2)}$ are satisfying the following conditions :

$$\lim_{s_{(1)} \to \infty} (\Psi^{\alpha}\Psi_{\alpha}) = 0, \tag{56}$$

And,

$$\lim_{\tau \to \infty} (\Phi_{\alpha}\Phi^{\alpha}) = 0. \tag{57}$$

Special Cases:

[1]Non-Geodesic Stability conditions: Higher Dimensions

In an approach to examine stability problems as a direct application of [18] .

Let us obtain the solution of the equation $\Psi^{A}(S)$ as exerted from the deviation equation in a given interval [A,B] , such that $\Psi^{A}(S)$ behave monotonically. Thus, such quantities describe the deviation vector may be sensors for measuring the stability in the following way

$$\hat{q} \stackrel{def.}{=} \lim_{S \to B} \sqrt{\Psi^{A}\Psi_{A}}. \tag{58}$$

If $\hat{q} \to \infty$ then the system is unstable, otherwise it is always stable.

where C^{A} are constants and $f(S)$ is a function known from the metric. If $f(S) \to \infty$, the system becomes unstable otherwise it is stable.

[2] Non-geodesic Stability conditions in Bi-metric Theories : On the Halos

For case

$$\frac{d\tau}{ds} \neq 0$$

The solution of the set deviation equations (...) and (...) are

$$\Psi a^{\alpha} = C_1^{\alpha} f(S) \tag{59}$$

And,

$$\Phi a^{\alpha} = C_2^{\alpha} f(S) \tag{60}$$

We must obtain two stability conditions in the following way:

$$q \stackrel{def.}{=} \lim_{s\to b} \sqrt{\Psi^{\alpha}\Psi_{\alpha}}. \tag{61}$$

and

$$\hat{q} \stackrel{def.}{=} \lim_{\tau\to b} \sqrt{\Phi^{\alpha}\Phi_{\alpha}}. \tag{62}$$

If $q \to \infty$ or $q \to \infty$ then the system is unstable, otherwise it is always stable.

[3] Non-geodesic Stability conditions in Bi-metric Theories : At the Accretion Disk

For case

$$\frac{d\tau}{ds} = 0$$

we have only one stability condition; condition as there exists one family of deviation equations $\tilde{\Psi}^{\alpha}$

$$\tilde{\Psi}^{\alpha}\tilde{C}^{\alpha} f(\tilde{S})$$

in which

$$\tilde{q} \stackrel{def.}{=} \lim_{\tilde{S}\to b} \sqrt{\tilde{\Psi}^{\alpha}\tilde{\Psi}_{\alpha}}.$$

thus, the stability condition becomes $\tilde{q} \to \infty$ then the system is unstable, otherwise it is always stable. Also, in a similar way , the case of diople moment trajectoiies as express in GR, we may have two simultaneous conditions one for linear passive momentum, as similar as for spinning particle with precession [32], and the other is for evolution deviation equation. Such a previous condition in case of its analogue in Bimetric theory such that $\frac{ds}{d\tau} = 0$ we obtain four conditions due to existence of twin matter equations. But it reduces to two conditions if we apply $\frac{ds}{d\tau} \neq= 0$.

5 Discussion and Conclusion

Dark matter may be regarded as a fluid with different characteristics behavior based on its position from the source of the gravitational field.

In view of the above mentioned speculation, we may give rise to emphasize previous ideas, such as of activated as dipolar fluid in the halo. The notation of a solely dipolar activity is analogous is inspired from the spinning motion of particles. Accordingly, in this study, we have developed the associated Lagrangian that presented simultaneously the linear equation and precession equation using a modified Bazanski Lagrangian. This can be employed in our case, by suggesting the motion dipolar moment be represented in linear momentum and evolution equations from one single Lagrangian [...].

Yet, such a tendency to express to include DM in the halo as dipolar particles is constraining this effect al galactic level, which is a violation to apparant observation that DM is also existed in the Universe as well as being felt to to associated incidences of excess of γ-ray radiation as an indicator of self annihilation DM-particles in the AGNs, as well as nearby neutron stars, binary pulsars. This can be revealed by applying Blanchet's approach of replacing the dipolar particles by dipolar fluids. In favor of this idea, the dipolar fluid may feel the interaction with dark energy and that itself quite reasonable due to the dominance of dark energy 74% while dark matter occupies 23% due to the well known observations, while baryonic matter is about 4%. Recently, due to ESA's Planck mission DM has been found to be 26%, which causes the other readings to be revised[11]; such an involvement may be consistent with an MOND-paradigm.

Now, the arising question is related to type of geometry and its associated gravitational field theory which is expressing this situation. Thus, it has been found that bi-metric theory of gravity can be regarded as a good representative to express this situation. However, the representation of bi-metric theory can be expressed differently from one region into another.

Accordingly it has led us to describe bi-metric theory behaving as bi-gravity ghost free massive theory at the galactic level to explain the behavior of DM as twin matter in a fluid, and near by the center of strong gravitational fields the conformal version is presented to be matched with Verozub's version of bi-metric theory of gravity. Owing to the equation of motion, it is vital to examine the stability of these regions, by solving the geodesic deviation equations, due to inter-relation between geodesic deviation equation and stability conditions.

In the present work, we extend Wanas-Bakry method [10] to include strong gravitational systems rather than its application on planetary systems. The advantage of this method is its invariance of coordinate systems.

Equations of relativistic hydrodynamics are expressed as non-geodesic equations, with taking into account that the mass excess term due to the existence of dark matter. Such a speculation, is an extension to similar vision by Kahil and Harko (2009)[1], who presented its existence in terms of a Lagrangian having a scalar defining variable mass.

The problem of motion as described in the Riemanian geometry will be extended to be explained by different geometries, admitting non vanishing curvature and torsion simultaneously. This work will be expressed in our future work to emphasize the concept of geometrization of physics i.e. the significance of the existence of DM and DE may be expressed within some quantities related to the richness of these geometries. This is a step towards revealing the cloud of mystery which is always associated with the notations of DM and DE.

References

[1] M.E. Kahil, M.E. and T. Harko , Mod.Phys.Lett.A24:667; arXiv:0809.1915 (2009).
[2] L. Blanchet and A. Le Tiec, Phys. Rev. D. **78**, 024031; arXiv 0804.3518 (2008)

[3] S.L. Bazanski J. Math. Phys., **30**, 1018 (1989).
[4]M.E.Kahil , J. Math. Physics **47**,052501 (2006).
[5]C.M. Ho, D. Minic, and Y.J. Ng, Gen. Relt. Gravit. **43**, 2567(2011).
[6]L.V. Verozub *Space-time Relativity and Gravitation, Lambert, Academic Publishing.*(2015)
[7]L .Blanchet L. Heisenberg , NORDITA-2015-38, arXiv 1504.00870 (2015)
[8]S. Peirani and J. A. de Freitas Pacheco, Phys. Rev. D. **78**,024031; arXiv 0802.2041 (2008)
[9] M. Milgrom , Astrophys. J. **270**, 365 (1983); M. Milgrom Phys.Rev.D80:123536,2009 arXiv: 0912.0790 (2009)
[10]M. Milgrom Phys. Rev. D 89, 024027 (2014); arXiv:1308.5388 (2014)
M.Milgrom arXiv: 1404.7661 (2014)
[11] L. Iorio, Galaxies 1,(**1**),6; arXiv: 1304.6396 (2013)
[12] I.B. Khiriplovich, Int. J.Mod Phys.,D **16**,1475 (2007)
[13]N. Rosen Gen. Relativ. and Gravit., **4**, 435 (1973).
N. Rosen, Ann. Physics ,455 (1974).
[14] S.F. Hassan and Rachel A. Rosen , JHEP,**2012**, 126 ; arXiv 1109.3515 (2012)
[15] K. Kleids and N.K. Spyrou, Class. Quant. Grav. **17**,2965; arXiv: gr-qc/9905055 (2000)
[16] M.I. Wanas, and M.A Bakry, Proc. MG XI, Part C, 2131; arXiv gr-qc/0703092 (2008)
[17]P.Wesson , J. Math Phys, **43**,2423; arXix: gr-qc/0105059 (2002)
[18]L. Blanchet , Class. Quant Grav.,**24**,3541 ; arXiv: gr-qc/0609121 (2007)
[19] M. Heydrai-Fard, M. Mohseni, and H.R. Sepanigi, H.R. (2005) Phys. Lett. B, **626**, 230.
[20] Magd E. Kahil Grav. Cosmol. **(24)**, 83 (2018); arXiv 1701.04136 (2018)
[21] B.C. Bromly, ApJs **197**,2; arXiv: 1112.2355 (2011)
[22]S. Hossenfelder arXiv: 0807.2838 (2008).
[23]L .Blanchet and L. Heisenberg, Phys Rev. D **96**,083512; arXiv 1701.07747 (2017)
[24] J.W.Moffat arXiv: hep-th/0208122 (2002)
[25]Y. Akrami, T. Kovisto, and A.R. Solomon arXiv.1404.0006 (2014)
[26] K. Aoki and K. Maeda arXiv: 1409.0202 (2014)
[27]Magd E. Kahil, Gravitation and Cosmology, Vol 23, 70 (2017).
[28]J.D. Bekenstein arXiv: gr-qc/9211017 (1992).
[29] R.C. Tolman, *Relativity, Thermodynamics and Cosmology*, Oxford Univ. Press (1934)
[30] M.I. Wanas , Astrophys. Space Sci., **127**,21 (1989)
[31] M.I. Wanas and M.A. Bakry, Astrophys. Space Sci., **228**,239
[32]Magd E. Kahil Odessa Astronomical Publications, vol **28/2**, 126 (2015)
[33] M. Roshan, Phys.Rev. D**87**,044005 (2013); arXiv 1210.3136

Appendix (A)

The Papapertrou Equation in General Relativity: Lagrangian Formalism

It is well known that equation of spinning objects in the presence of gravitational field have been studied extensively []. This led us to suggest its corresponding Lagrangian formalism , using a modified Bazanski Lagrangian [20], for a spinning and precessing object and their corresponding deviation equation in Riemanian geometry in the following way

$$L = g_{\alpha\beta} P^{\alpha} \frac{D\Psi^{\beta}}{Ds} + S_{\alpha\beta} \frac{D\Psi^{\alpha\beta}}{Ds} + F_{\alpha}\Psi^{\alpha} + M_{\alpha\beta}\Psi^{\alpha\beta} \qquad (A.1)$$

where

$$P^{\alpha} = mU^{\alpha} + U_{\beta}\frac{DS^{\alpha\beta}}{DS}.$$

Taking the variation with respect to Ψ^{μ} and $\Psi^{\mu\nu}$ simultaneously we obtain

$$\frac{DP^{\mu}}{DS} = F^{\mu}, \qquad (A.2)$$

$$\frac{DS^{\mu\nu}}{DS} = M^{\mu\nu} \qquad (A.3),$$

where P^{μ} is the momentum vector, $F^{\mu} = \frac{1}{2} R^{\mu}_{\nu\rho\delta} S^{\rho\delta} U^{\nu}$, and $R^{\alpha}_{\beta\rho\sigma}$ is the Riemann curvature, $\frac{D}{Ds}$ is the covariant derivative with respect to a parameter S,$S^{\alpha\beta}$ is the spin tensor, $M^{\mu\nu} = P^{\mu}U^{\nu} - P^{\nu}U^{\mu}$, and $U^{\alpha} = \frac{dx^{\alpha}}{ds}$ is the unit tangent vector to the geodesic.
Using the following identity on both equations (1) and (2)

$$A^{\mu}_{;\nu\rho} - A^{\mu}_{;\rho\nu} = R^{\mu}_{\beta\nu\rho} A^{\beta}, \qquad (A.4)$$

where A^{μ} is an arbitrary vector. Multiplying both sides with arbitrary vectors, $U^{\rho}\Psi^{\nu}$ as well as using the following condition [19]

$$U^{\alpha}_{;\rho}\Psi^{\rho} = \Psi^{\alpha}_{;\rho}U^{\rho}, \qquad (A.5)$$

and Ψ^{α} is its deviation vector associated to the unit vector tangent U^{α}. Also in a similar way:

$$S^{\alpha\beta}_{;\rho}\Psi^{\rho} = \Phi^{\alpha\beta}_{;\rho}U^{\rho}, \qquad (A.6)$$

one obtains the corresponding deviation equations [34]

$$\frac{D^2\Psi^{\mu}}{DS^2} = R^{\mu}_{\nu\rho\sigma} P^{\nu} U^{\rho} \Psi^{\sigma} + F^{\mu}_{;\rho}\Psi^{\rho}, \qquad (A.7)$$

and

$$\frac{D\Psi^{\mu\nu}}{DS} = S^{\rho[\mu} R^{\nu]}_{\rho\sigma\epsilon} U^{\sigma}\Psi^{\epsilon} + M^{\mu\nu}_{;\rho}\Psi^{\rho}. \qquad (A.8)$$