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Reduced Complexity Spatial Modulation Transmit Precoding for PSK Constellation

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Abstract

Spatial modulation (SM) conveys extra data by selecting the transmit antenna. This makes SM prone to channel irregularities like multipath Raleigh fading. Hence, employing and optimizing a transmit precoder (TPC) that matches the channel can enhance the SM bit error rate performance by increasing the Euclidean distance (ED) among all possible received vectors. However, it is common that optimization algorithms endure high complexity. Focusing on M-PSK constellation, and by reducing the number of Euclidean Distance constraints, we cut the complexity by nearly a factor of M. This is a significant reduction for high order constellations with a large value of M. This concept is shown to benefit any TPC optimization algorithm for SM and its variants. To further shrink the complexity, we introduce an optimization algorithm that minimizes the sum of the exponentials of negative EDs. The paper shows that the complexity can be reduced significantly without loss in performance.

Keywords: Augmented Lagrangian, quasi-Newton, optimization, maximum minimum Euclidean distance, precoding, spatial modulation

1. Introduction

Recently, there has been an increasing interest in Spatial Modulation (SM) techniques for future wireless communication systems [1]. The recent literature shows innovations and new variants of SM that improve its potential [2]. Thanks to its basic working mechanism, SM provides many advantages, including higher energy efficiency, lower detection complexity, lower synchronization requirements, and no inter-channel interference [3]. The basic SM works as follows. Consider a system with N_t transmit antennas (TAs), where N_t is a power of 2, and M-ary modulation. At every transmission instant $\log_2 N_t + \log_2 M$ bits are transmitted. The first $\log_2 N_t$ bits select the active antenna index. The last $\log_2 M$ bits select one symbol from M-ary constellation for transmission from this antenna. Other variants of SM are proposed in the literature to increase the transmission rate at the cost of complexity [2].

For successful detection of the SM signal the receiver needs to decide which antenna is activated. This requires the Channel State Information (CSI) from the N_t TAs to be unique. Although the elements of the CSI may be statistically independent, they can be instantaneously similar, leading to errors in detection. To circumvent this problem, transmit precoding is typically used [2] [4] [5]. A Transmit Precoder (TPC) may be selected from a predefined codebook that is known to both the transmitter and the receiver [6] [7]. Alternatively, the TPC can be optimized for the CSI, providing better performance. Most optimization algorithms require CSI at the transmitter (CSIT). This is facilitated through feedback [8]. However, they endure high complexity at large M and N_t. Therefore, this paper presents a technique to reduce the complexity of the TPC optimization algorithm. Here, we focus on M-ary PSK constellation due to its advantages in SM systems [4] [9] [10] and its potential for reducing the complexity. However, the presented technique can be used for M-ary QAM but with more complexity.

A comprehensive survey of TPC optimization algorithms is available in [2] [4]. Here, we review the directly related literature. The work in [11] studies two objective functions for optimizing the TPC. The first is to maximize the Minimum Euclidean Distance (MED) while constraining the total transmit power. The second is to minimize the total transmit power with a guaranteed MED. The two approaches provide identical Bit Error Rate (BER) that is lower than the BER without precoding. Reference [9] compares the BER performance when maximizing the MED (similar to the first approach in [11]) to directly minimizing the BER. The latter approach provides better performance and lower computational complexity than [11]. To reduce the computational complexity further, the work in [12] employed a low-complexity algorithm to minimize the total transmit power with a guaranteed MED (the second approach in [11]). The same BER of [11] is realized.

Other TPC calculation methods avoid employing iterative optimization algorithms and the associated complexity. In [13] a Zero-Forcing based TPC is employed, while in [14] a Minimum Mean Square Error TPC is proposed. Both methods provide a closed form of the TPC. However, the BER performance is worse. Hence, in this paper we focus on TPC optimization algorithms and attempt to reduce their complexity.

Most of the existing work reduces complexity through adapting an efficient optimization algorithm and manipulating the objective function and/or its constraints. The complexity of the algorithms depends mainly on the number of SM Euclidean Distances (EDs). In [9], [11], [12] and many other papers the number of EDs is $MN_t\left(MN_t-1\right)/2$. This number is true for any arbitrary M-ary constellation without benefitting from its symmetry. The work in [15] shows that for M-ary PSK the number of active EDs is only $N_t+MN_t\left(N_t-1\right)/2$. This paper reduces this number to $N_t+N_t\left(N_t-1\right)/2$, which significantly reduces complexity without loss in BER performance.

2. System and Channel Models

The system model is shown in **Error! Reference source not found.**. The transmitter and receiver are equipped with Nt and Nr antennas, respectively. At the transmitter, the first log2 M bits select an M-PSK symbol $s_m=\exp\left(2\pi m/M\right)$, m=0, 1, …, M-1, from the constellation. The next log2 Nt bits activate one TA. The transmitter and receiver employ CSI to optimally calculate the Nt×1 complex TPC vector p (p may also be fed-back from the receiver).

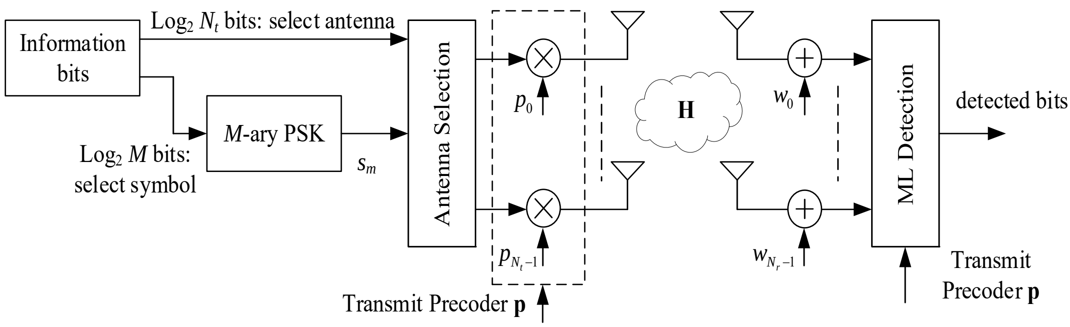

Figure 1: SM system model with precoding. The TPC vector p is calculated using an optimization algorithm

Each TA is multiplied by an element pk from the vector p. The signal is transmitted through a frequency non-selective Rayleigh fading channel H (the Nr×Nt CSI matrix). Entries hi,j of H are independent complex Gaussian random variables with zero-mean and unit variance. Let hk be the k'th column of H, $k = 0, 1, \cdots N_t - 1$. If symbol sm is transmitted from the k'th TA, the transmit vector is $\mathbf{v}_{m,k} = p_k s_m \mathbf{h}_k$ and the received signal is:

$$\mathbf{r} = \mathbf{v}_{m,k} + \mathbf{w} \quad . \qquad (1)$$

The (Nr×1) vector w is the complex additive white Gaussian noise whose elements are independent with zero-mean and variance 1/γ , where γ is the average signal to noise ratio (SNR) per receive antenna. The receiver employs Maximum Likelihood (ML) detection [16] as follows:

$$\left\{\hat{m}, \hat{k}\right\} = \min_{\substack{k=0,1,\ldots,N_t-1 \\ m=0,1,\ldots,M-1}} \left(\left\| \mathbf{r} - \mathbf{v}_{m,k} \right\|^2 \right) \quad . \qquad (2)$$

Other reduced complexity detection detectors are available in the literature[17]. However, we employ an optimum detector so that the BER comparison with the literature is attributed only to the proposed optimization algorithm. Let the MN_t transmit vectors vm,k be indexed as $\mathbf{v}_i \equiv \mathbf{v}_{k,m}$, with $i = m\,N_t + k$ and denote the number of bit differences between transmit vectors vi and vj as Bi,j. Conditioned on H, and using the Q-function, a union bound for the BER is calculated using the EDs among all possible transmit vectors as [16]:

$$P_e \big| \mathbf{H} \le \frac{2}{MN_t \log_2\left(MN_t\right)}$$

$$\sum_{i=0}^{MN_t-1} \sum_{j>i}^{MN_t-1} B_{i,j}\; Q\left(\sqrt{\frac{\gamma}{2} \left\| v_i - v_j \right\|^2} \right) \quad , \ (3)$$

3. Reducing the Number of Euclidean Distances

To reduce the BER bound of (3), TPC optimization algorithms iteratively increase the EDs $\left\| v_i - v_j \right\|^2$ by updating the TPC vector p. Depending on the problem formulation, the EDs are present in the objective function or constraints. Since the number of possible transmit vectors is MN_t, the optimization algorithms in [9], [11], [12] and many others took the number of EDs as $MN_t\left(MN_t-1\right)/2$. If we limit the scope to M-PSK this number can be significantly reduced. For M-PSK, the EDs (denoted below as di) are given by:

$$d_i=\left\|\mathbf{v}_{m,k}-\mathbf{v}_{n,l}\right\|^2=\left|p_k\right|^2\left\|\mathbf{h}_k\right\|^2+\left|p_l\right|^2\left\|\mathbf{h}_l\right\|^2$$

$$-2\left|\mathbf{h}_k^H\mathbf{h}_l p_k^* p_l\right|\cos\left(\theta_{k,l}+\frac{2\pi f}{M}\right) \quad , (4)$$

Where $f=n-m$, $\theta_{k,l}=\angle\mathbf{h}_k^H\mathbf{h}_l p_k^* p_l$, and $\angle x$ is the angle (0 to 2π) of the complex number x. The EDs can be between transmit vectors from the same antenna (i.e., $l=k$ and $n\neq m$), or from different antennas (i.e., $l\neq k$). From (4), the MED between transmit vectors from the same antenna is given by:

$$\min_{m,n}\left\|\mathbf{v}_{m,k}-\mathbf{v}_{n,k}\right\|^2=4\left|p_k\right|^2\left\|\mathbf{h}_k\right\|^2\sin^2\frac{\pi}{M} \quad . (5)$$

Hence, for $l=k$ the algorithm needs only to increase the Nt MEDs given by (5).

Switching attention to EDs between transmit vectors from different antennas, the work in [15] noted that (4) provides a total of $N_t+MN_t\left(N_t-1\right)/2$ unique EDs. They can be calculated by setting $l>k$ and $f=0,1,\cdots,M-1$. Hence, according to [15], the optimization algorithms needs to increase these EDs and, consequently, the are included in the objective function and/or the constraints. In the remainder of this paper, we refer to this number as the full EDs.

In this paper we lower the complexity by reducing of the number of EDs further. By careful examination of (4) when $l>k$, we notice that for any antenna pair (k, l) there are M unique EDs corresponding to f=0, 1, …, M-1. Their minimum is easily found by maximizing the cos(.) in (4) by selecting the value of f for this pair as:

$$f_{k,l}=M-round\left(\frac{\theta_{k,l}}{2\pi/M}\right) \quad , \quad l>k \quad . (6)$$

Hence, for $l>k$ the optimization algorithm needs only to increase the EDs corresponding to the $N_t\left(N_t-1\right)/2$ antennas pairs (k, l) given by (6). After updating the TPC vector p, each iteration of the optimization algorithm uses (6) to select a single ED from (4) for each antenna pair. Together with the Nt EDs calculated from (5), the total number of EDs becomes $N_t + N_t(N_t - 1)/2$. In the remainder of this paper, we refer to this number as the reduced EDs. Compared to the case of full EDs, a considerable reduction in complexity is achieved. Moreover, with reduced EDs the same BER performance is achieved and the optimization algorithm converges faster.

4. Transmit Precoder Optimization Algorithm

It is shown in [9] that minimizing the BER provides better performance than maximizing the MED. To reduce complexity, we adopt a simplified version of (3) as the objective function. We keep only the $N_t + N_t(N_t - 1)/2$ reduced EDs terms and drop the factor Bi,j. Next, we invoke the Chernoff bound of the Q function as $Q(x) \le \exp(-x^2/2)$. Lastly, to avoid re-optimization for each SNR we set the exponent $\beta = \gamma_o/4$, with γo being a target SNR. The optimization problem becomes:

$$\begin{aligned} \min_{\mathbf{p}} \; A = \sum_{i=\{S\}} \exp\left(-\beta\left(d_i - d_{\min}\right)\right) \\ s.t. \quad \|\mathbf{p}\|^2 = N_t \end{aligned} \quad , \qquad (7)$$

where S is the set of reduced EDs. The first Nt EDs in the set {S} (i.e., d0, to d_{N_t-1}) are calculated from (5). The remaining $N_t(N_t - 1)/2$ EDs are calculated from (4) and (6). The factor $d_{\min} = \min(d_i)$ is included to avoid a diminishing summation in (7). To solve (7) we use the augmented Lagrangian algorithm [18] due to its low complexity and fast conversion [12]. Hence, (7) is converted into the unconstrained optimization:

$$\begin{aligned} \min_{\mathbf{p}} \; L(\mathbf{p}) = \sum_{i=\{S\}} \exp\left(-\beta\left(d_i - d_{\min}\right)\right) \\ -\lambda\left(\|\mathbf{p}\|^2 - N_t\right) + \frac{\mu}{2}\left(\|\mathbf{p}\|^2 - N_t\right)^2 \end{aligned} \quad , \qquad (8)$$

where λ is the Lagrange multiplier and μ is the penalty parameter [18]. The optimization algorithm is shown in Algorithm 1. We use initialization parameters from [12].

Algorithm 1: Augmented Lagrangian Optimization

Initialization: p0=all ones, λ=0.5 , μ=10, ρ=2, n=0

Pre-calculate: $\left\|\mathbf{h}_k\right\|^2$, $\mathbf{h}_k^H \mathbf{h}_l$, k, l=0, 1, .., Nt-1, l > k

Repeat 1 to 4 till termination:

1) Update: $\mathbf{p}_{n+1} = \arg\min_{\mathbf{p}} L\left(\mathbf{p}_n\right)$ and get all di

2) Find $MED_{n+1} = d_{\min} = \min\left(d_i\right)$

3) Update: $\lambda = \lambda - \mu\left(\left\|\mathbf{p}_{n+1}\right\|^2 - N_t\right)$

4) Update: $\mu = \rho\, \mu$

Termination: if MEDn+1 < MEDn and n > 1 then output pn

An unconstrained optimization method is needed in step 1. Similar to [12] we employ the quasi-Newton method since it is efficient and robust [18] [19]. This method requires the gradient of the objective function (8), which can be written as:

$$\begin{aligned}\mathbf{g} &= \nabla_{\mathbf{p}} L\left(\mathbf{p}\right) \\ &= -\beta \sum_{i=\{S\}} \nabla_{\mathbf{p}} d_i\, \mathrm{e}^{-\beta\left(d_i - d_{\min}\right)} - 2\left(\lambda - \mu\left(\left\|\mathbf{p}\right\|^2 - N_t\right)\right)\mathbf{p}\end{aligned} \quad . (9)$$

The set {S} in (9) includes an equal number of EDs for all antennas. Hence, the gradient g is confidently used in the optimization algorithm to update the TPC vector p for all antennas. To calculate (9) we need the vectors $\nabla_{\mathbf{p}} d_i$. The first Nt distances in {S} come from (5). Define the Nt×1 complex vectors $\mathbf{x}_i = \nabla_{\mathbf{p}} d_i$, $i = 0, 1, \ldots, N_t - 1$. Using (5) we get $\mathbf{x}_i = \left[00\cdots x_i \cdots 00\right]^T$. Hence, xi has a single non-zero element at the i'th entry, which is given by:

$$x_i = 8 p_i \left\|\mathbf{h}_i\right\|^2 \sin^2\left(\pi/M\right) \quad . \qquad (10)$$

The remaining distances in {S} come from (4). Define the Nt×1 vectors $\mathbf{y}_i = \nabla_{\mathbf{p}} d_i$, $i = N_t, N_t + 1, \cdots, N_t\left(N_t - 1\right)/2 - 1$. Using (4) and (6) we find that $\mathbf{y}_i = \left[00\cdots y_k 0\cdots 0 y_l \cdots 00\right]$, i.e., yi has only two non-zero elements at the k'th and l'th entries. The k'th and l'th entries in yi are given from (4) and (6) as:

$$
\begin{aligned}
y_k &= 2\,p_k\,\|\mathbf{h}_k\|^2 - 2\mathbf{h}_k^H\mathbf{h}_l\,p_l\,\exp\left(j2\pi\,f_{k,l}/M\right)\\
y_l &= 2\,p_l\,\|\mathbf{h}_l\|^2 - 2\mathbf{h}_l^H\mathbf{h}_k\,p_k\,\exp\left(-j2\pi\,f_{k,l}/M\right)
\end{aligned}
\quad . \qquad (11)
$$

With all parts of (9) ready, we now describe the quasi-Newton method. This method requires the calculation of the inverse of the Hessian matrix of the objective function (8). The BFGS algorithm is employed to avoid matrix inversion. However, the BFGS algorithm is defined for a real matrix only. Hence, similar to [12], in the quasi-Newton method we convert the TPC vector p and the gradient vector g into their equivalent 2Nt×1 real vectors. For any N×1 complex vector x, its equivalent 2N×1 real vector is $\overline{\mathbf{x}} = \left(real\left(\mathbf{x}^T\right), imag\left(\mathbf{x}^T\right)\right)^T$.

Algorithm 2: Quasi-Newton method for step 1 in Algorithm 1

Initialization: $\mathbf{B} = \mathbf{I}_{2N_t}$, n=0, get p0 and dmin from Algorithm 1, and find $\mathbf{g}_0 = \nabla_{\mathbf{p}} L\left(\mathbf{p}_0\right)$ using (9)

Repeat:

Search direction: $\overline{\mathbf{d}} = -\mathbf{B}\,\overline{\mathbf{g}}_n$

Update TPC: $\overline{\mathbf{p}}_{n+1} = \overline{\mathbf{p}}_n + \alpha_n \overline{\mathbf{d}}$

Update g: $\mathbf{g}_{n+1} = \nabla_{\mathbf{p}} L\left(\mathbf{p}_{n+1}\right)$ using (9)

BFGS update of inverse Hessian:

$\mathbf{a} = \alpha_n \overline{\mathbf{d}}$, $\mathbf{b} = \overline{\mathbf{g}}_{n+1} - \overline{\mathbf{g}}_n$

$$
\mathbf{B} = \mathbf{B} + \left(1 + \frac{\mathbf{b}^T\mathbf{B}\mathbf{b}}{\mathbf{b}^T\mathbf{a}}\right)\frac{\mathbf{a}\mathbf{a}^T}{\mathbf{b}^T\mathbf{a}} - \frac{\mathbf{a}\mathbf{b}^T\mathbf{B} + \mathbf{B}\mathbf{b}\mathbf{a}^T}{\mathbf{b}^T\mathbf{a}}
$$

Termination: $\max\left(\mathbf{a} \div \left(1 + \overline{\mathbf{p}}_{n+1}\right)\right) < \varepsilon$

In step 2 of Algorithm 2, the step size αn is found using the line search algorithm described in section 2.6 of [20]. dmin is input from algorithm 1 and fixed during the iterations of Algorithm 2. The symbol (÷) in the Termination step indicates point by point division. The termination threshold is ε=1.0E-5. Using Algorithms 1 and 2, the optimum TPC vector p is calculated and employed in the transmission (1) and reception (2).

5. Numerical results and Discussion

This section provides the complexity and BER performance of the proposed method. Simulation results are averaged over 5×105 independent channel realizations. In

Figure 3 we use β=55. In the other figures with Nt=8 the value of β is as follows:

N_r	M=4	M=8	M=16
2	11	22	44
4	5.5	11	22

5.1. Complexity Analysis

Complexity depends on the long-term order of the number of complex multiplications and the speed of convergence. We ignore the complexity of the trigonometric since it is efficiently calculated using the CORDIC algorithm. Also, similar to [12] we ignore the complexity of the line search algorithm since, by definition, it is much smaller than the optimization algorithm. The complexity of the one-time calculation of $\|\mathbf{h}_k\|^2$ and $\mathbf{h}_k^H\mathbf{h}_l$ is $\mathrm{O}(N_rN_t)+\mathrm{O}(N_rN_t^2)$. Let K_1 and K_2 be the number of iterations in Algorithm 1 and Algorithm 2, respectively. The complexity of Algorithm 1, excluding the quasi-Newton method in step 1, is $\mathrm{O}(K_1N_t)$ to calculate $\|\mathbf{p}_{n+1}\|^2$ in step 3. Note that the EDs used in minimum selection in step 2 are calculated in the quasi-Newton method. The complexity of Algorithm 2 is $\mathrm{O}(K_1K_2N_t^2)$ since (5) and (10) are calculated Nt times per iteration, while (4) , (6) and (11) are calculated $N_t+N_t(N_t-1)/2$ times per iteration. Also, the complexity of calculating the one-time inverse Hessian matrix is $\mathrm{O}(N_t^2)$. Hence, the total complexity order is given by:

$$\mathrm{O}(N_rN_t)+\mathrm{O}(N_rN_t^2)+\mathrm{O}(K_1N_t)+\mathrm{O}(K_1K_2N_t^2) \quad . \quad (12)$$

If full EDs are used, the last term of (12) would be $\mathrm{O}(K_1K_2MN_t^2)$.

Table 1. Complexity Order with N_t=8 AND N_r=2

	QPSK	8-PSK	16-PSK

Average K_1: *full EDs*	3.3579	3.3082	2.8693
Average K_1: *reduced EDs*	3.6391	3.4681	2.8441
Average K_2: *full EDs*	53.5160	44.2325	38.1295
Average K_2: *reduced EDs*	38.4152	36.4811	28.9986
Complexity: *full EDs*	46174	75091	112198
Complexity: *reduced EDs*	9120	8269	5445
[9], Table II, min-BER	61440	245760	983040
[12], Table I, AL method	276096	1065600	4223616

Table 1 shows the complexity reduction from full EDs to reduced EDs. Also, it shows that the complexity is much lower than that of [9] and [12], where the complexity formulas in their respective references used. The average number of iterations in Algorithm 1 (K1) is almost the same in full EDs and reduced EDs. However, the average number of iterations in Algorithm 2 (K2) is lower, providing lower complexity.

Figure *1* shows an example of the probability mass function of K2 for both the full EDs and Reduced EDs methods.

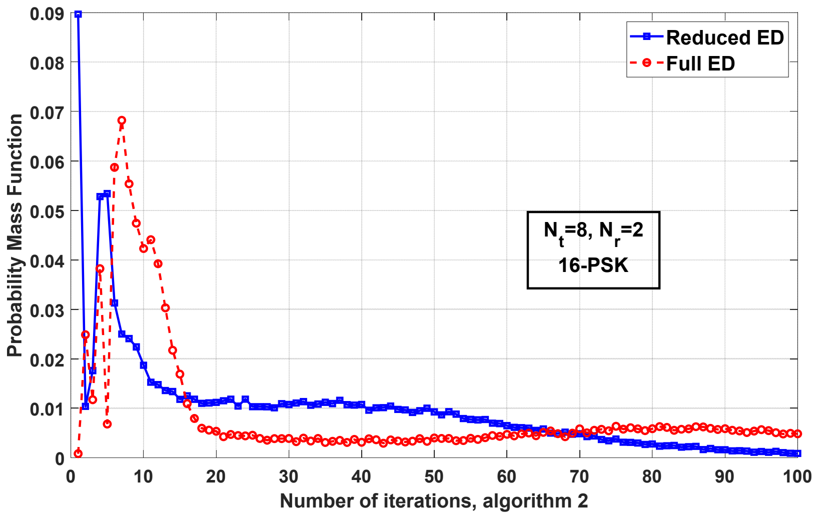

Figure 1. Probability mass function of the number of iterations in algorithm 2 (K_2) for 16-PSK, N_t=8, N_r=2 (line style is used instead of histogram)

5.2. BER Performance

We found by simulation that the BER of full EDs and reduced EDs is identical. Hence, we show BER with reduced EDs.

Figure 2 compares the BER of the proposed reduced EDs algorithm with [9] (minimum BER) and [12] (maximum MED) for QPSK with Nt=8 and Nr=2. At low SNR, the proposed algorithm is close to [12]. As the SNR increases, it gets closer to [9] and even slightly better at high SNR.

Figure 2 also shows the effect of channel estimation error. The erroneous CSI is given by $\mathbf{H}_e = \mathbf{H} + \Delta\mathbf{H}$, where the elements of the matrix $\Delta\mathbf{H}$ are independent complex Gaussian with zero mean and variance $1/\gamma$ [9].

We show the BER when He affects the optimization algorithm only, and when it also affects the ML receiver. The performance of SM without precoding is used for reference. The proposed optimization algorithm is still advantageous compared to SM.

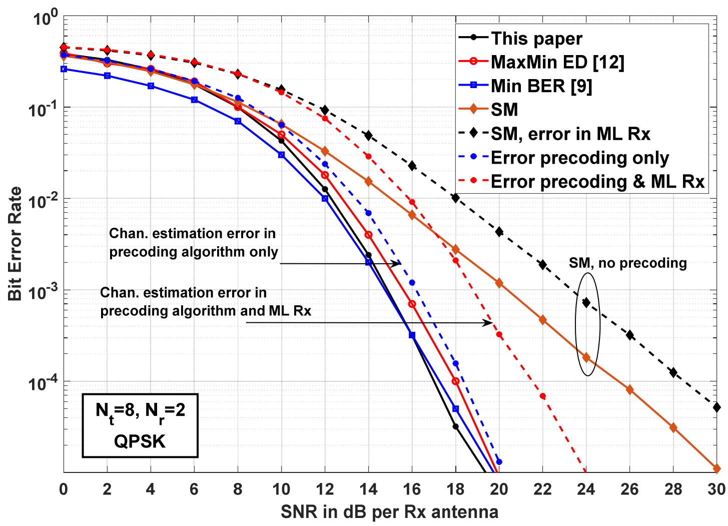

Figure 2. BER comparison of the proposed algorithm with [9] and [12] for QPSK, N_t=8, N_r=2. Dashed lines are with channel estimation error.

Next, we explore high order modulation schemes with a small number of TAs.

Figure 3 presents the performance of the proposed algorithm for 16-PSK with Nt=4 and Nr=2. Similar to

Figure 2, at low SNR the performance is close to [12]. As the SNR increases it gets closer to [9]. The figure also shows the BER of 16-QAM using the same algorithm (yet higher complexity). For the used set of parameters there is no gain in using 16-QAM compared to 16-PSK.

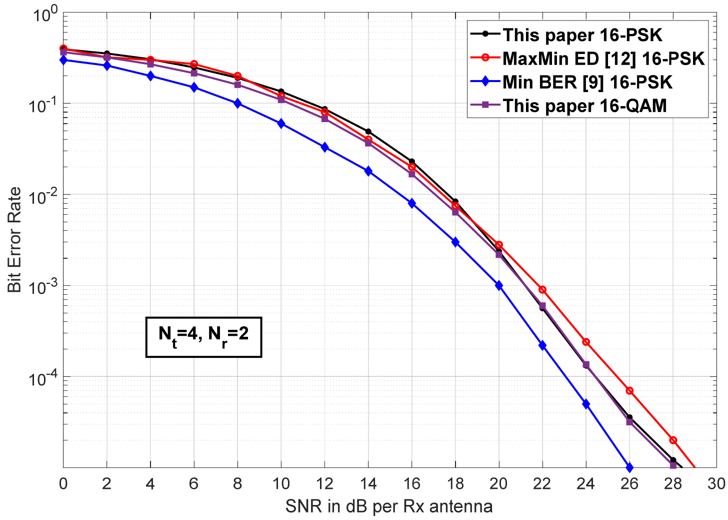

Figure 3. BER comparison of the proposed algorithm for 16-PSK and 16-QAM with [9] and [12]. N_t=4, N_r=2

To explore larger Nr,

Figure *4* presents the performance of the proposed algorithm for 16-PSK with Nt=8 and Nr=2 and 4. The proposed system consistently provides a performance better than [12] for both Nr=2 and Nr=4. We also show the BER for 16-QAM. Only when Nr=4, there is a small advantage of 16-QAM over 16-PSK. However, it does not seem to justify the added complexity.

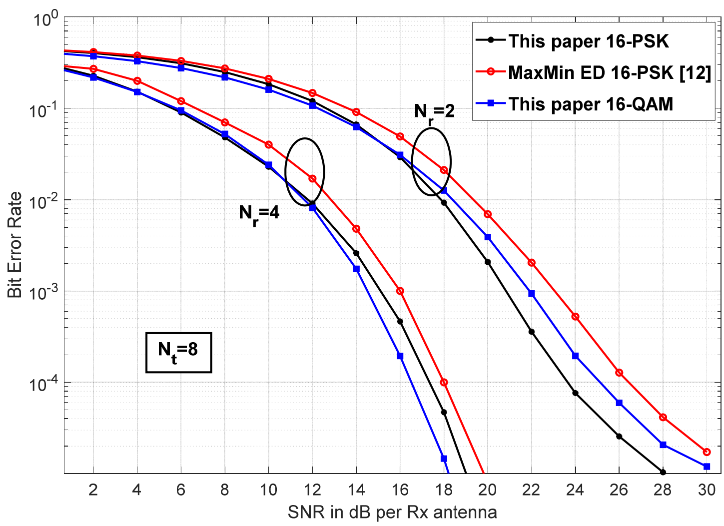

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Figure 4. BER comparison of the proposed algorithm for 16-PSK and 16-QAM with [12]. Nt=8, Nr=2 and 4

From the above BER results we conclude that the proposed reduction in the EDs does not cause any loss in BER. To the contrary, the BER is always better than [12].

6. Conclusion

This paper introduces a method of lowering the complexity of transmit precoder optimization in SM systems, employing M-PSK constellation. This is achieved by selecting one Euclidean distance per antenna pair, instead of all M Euclidean distances. We implement the proposed concept using an optimization problem that minimizes the sum of the exponentials of negative Euclidean distances. The optimization problem is solved with augmented Lagrangian algorithm. Our simulation shows equivalent or better BER than other published results, but with much lower complexity. Hence, the complexity reduction method of this paper is a candidate for all SM systems and its variants employing M-PSK constellation.

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