



## *On the Derivative Backoff Problem in PID Controllers*

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### **Abstract**

The Proportional-Integral-Derivative (PID) is by far the most common controller in process industries. In practice, a problem with PID controllers may arise when the controlled process variable (PV) saturates. At this point, the error, i.e., the difference between the set point (SP) and PV becomes constant, and so the derivative control action becomes zero or backs off. This leads to a sudden increase in the total controller output and as a result the process variable moves above its limit showing larger overshoot and settling time. To solve this problem, it is proposed to modify the PID controller action when the PV saturates. The modification is simply to multiply the derivative part by a suitable gain, transfer it to the integral part, and then the derivative part is set to zero. When the process output later becomes unsaturated, the derivative action is activated again. This technique is shown to reduce the overshoot, settling time, integral of absolute error (IAE), and works well in the presence of measurement noise. Although the optimal value of the gain depends on the size of disturbance which is not usually known, a fixed value of 2 is shown to be reasonable for most levels of disturbance.

**Keywords:** PID control, derivative backoff, process variable saturation.

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## 1. Introduction

Control systems play indispensable role in process industries ensuring product quality, safety of operators, equipment, environment protection, and maximizing profit. Over the years, diverse types of controllers ranging from the simple proportional-integral-derivative (PID) controller to the more sophisticated adaptive, robust, and optimal controls, are developed. Despite its simplicity, PID control is the most used controller in process industries [1, 2, 3] as it achieves acceptable performance while being well-understood by process operators. For this reason, enormous research efforts are devoted to improving PID controller parameter tuning [4, 5] as well as automatic tuning [6, 7]. In addition, the two-degrees of freedom configuration is proposed to achieve both good set point and load disturbance rejection [8]. Several variants of the controller are devised to deal with nonlinear processes such as adaptive [9], nonlinear [10], and fractional-order PID controllers [11, 12]. Other efforts are devoted to treat implementation issues such as anti-windup mechanisms [13].

One of the main issues with PID controllers is the well-known integral windup problem that arises when the control signal hits a limit while the error is not zero. The consequence is that the integral part continues to grow above the value required at steady state leading to large overshoot and settling time in the process variable (PV). Among many techniques to solve this issue, conditional integration can be used, in which the integral action is stopped once the control signal saturates [13].

Recently, Theorin and Hagglund [14] pointed to another saturation problem associated with PID controllers, called derivative backoff. This problem occurs when a process variable itself saturates or hits one of its two limits (0% or 100%). This can occur, for example, when the set point is close to the limit and a sufficiently large disturbance occurs causing the process variable to overshoot above its upper limit (100%) or below its lower limit value (0%). At this instant, both the measured process variable and error are constant and so the derivative action becomes zero or backs off. This in turn results in a sudden increase in the control signal which drives the process variable away from set point leading to large overshoot and more time to return to the set point. The same problem can also appear in, e.g., active disturbance rejection controller (ADRC) [15]. To treat this problem, Theorin and Hagglund [14] proposed the following solution: when the process variable hits its limit, transfer, bumplessly, the derivative part value to the integral part and then set the derivative part to zero. When the process output later becomes unsaturated, the derivative action is activated again. This solution succeeded in reducing overshoot and settling time and works well in the presence of measurement noise.

In this paper, an enhanced variant of the approach by Theorin and Hagglund [14] is proposed. The idea is to multiply the derivative action by a certain gain,  $\alpha > 1$ , before transferring it to the integral part. This is shown to achieve less overshoot, settling time, and integral of absolute error (IAE). To choose the gain  $\alpha$ , an experiment is conducted to optimize the IAE for different levels of disturbance. Although, the value of optimal gain depends on how large the disturbance is, a constant gain of 2 is found to be suitable for most levels of disturbances.

The rest of the paper is organized as follows. First, in Section 2, the derivative backoff problem is described. Next, an improved solution to the problem is proposed in Section 3, along with several experiments to test and evaluate the performance of the method. In Section 4, the optimal selection of the gain  $\alpha$  is discussed. The robustness of the proposed method is tested through another experiment in Section 5. Finally, some conclusions are drawn in Section 6.

## 2. The derivative backoff problem

Consider the feedback control loop in Fig. 1, which contains a saturation element in the feedback path. The signals  $r, y, y_m, d$ , and  $u$  denote the set point, process variable to be controlled, measured process variable, load disturbance, and the control signal, respectively. The measured process variable  $y_m$  is related to the actual process variable  $y$  through the saturation element as follows:

$$y_m = \begin{cases} y_{min}, & \text{if } y \leq y_{min} \\ y, & \text{if } y_{min} \leq y \leq y_{max} \\ y_{max}, & \text{if } y \geq y_{max} \end{cases} \quad (1)$$

where  $y_{max}$  and  $y_{min}$  denote the maximum and minimum values for the process variable, respectively. The PID controller operates on the error,  $e = r - y_m$ , where the control signal,  $u$ , is calculated as a weighted combination of the error, its integral, and its derivative according to the following formula:

$$u(t) = K \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right), \quad (2)$$

where the parameters  $K, T_i$ , and  $T_d$  are the controller gain, integral time, and derivative time, respectively. In practice, the derivative term is filtered using a first order filter:

$$G_f(s) = \frac{T_d s}{\frac{T_d}{N} s + 1}, \quad (3)$$

where the derivative gain  $N$  is usually set to 10.

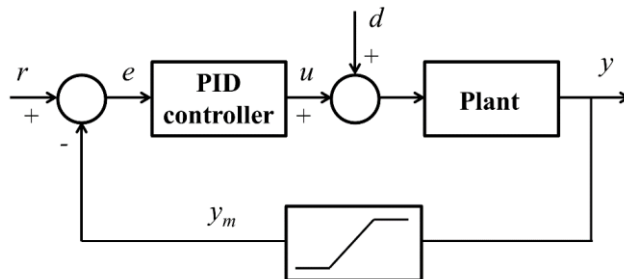


Fig. 1. The feedback control loop with saturation element in the feedback path.

To illustrate the derivative backoff problem, Theorin and Haggglund [14] considered the following process,

$$P(s) = \frac{1}{(s+1)^4}, \quad (4)$$

This process is controlled using a PID controller tuned using the M-constrained Integral Gain Optimization (MIGO) method with M, the maximum sensitivity function, set to 1.4 [2]. The plant given by Eq. (4) is representative of self-regulating processes commonly found in process industries. The process is discretized, using a zero-order hold (ZOH), with a sample time of 0.04 s. The controller parameters are  $K = 1.19$ ,  $T_i = 2.22$ , and  $T_d = 1.21$ . The purpose of these settings is to show that the derivative backoff problem could exist even if the PID controller is well tuned [14]. Furthermore, for this setup to reflect practical situations, it is imperative to investigate the effect of measurement noise. For this purpose, a Gaussian random noise of variance 0.01 is added to the measured process variable through the set up shown in Fig. 2, where the noise can cause false saturation of the process variable while being close to its upper or lower limit [14].

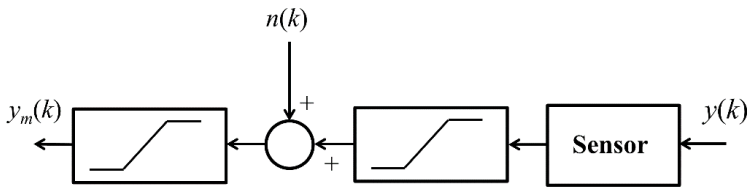


Fig. 2. Channel noise setup.

In the simulations to follow, the set point is 90%, i.e., very close to the limits of the process variable to put more focus on the problem. A step load disturbance of 40% is introduced at the process input. The response of the process variable as well as the control signal are shown in Fig. 3 with ordinary PID without any specific action taken against the derivative backoff. It is clear from Fig. 3 that the derivative backoff problem is profound; the actual process variable is driven away from set point and takes longer time to return back to the set point. To deal with this problem, Theorin and Haggglund [14] suggested an anti-backoff method, in which once the process variable hits a limit, the value of the derivative term moves to the integral part and then the derivative term is set to zero. This can be expressed as

$$\begin{aligned} \text{if } y_m = y_{max} \text{ or } y_{min}: \\ I = I + D, \\ D = 0, \end{aligned} \quad (5)$$

where I and D denote the integral and derivative controller actions, respectively. The response obtained using this method is also shown in Fig. 3 where it is clear that both overshoot and settling time are reduced thanks to the reduction of control signal (Fig. 3, bottom) when the process variable starts to saturate. Based on this idea, an improved variant of this scheme is proposed in the next section.

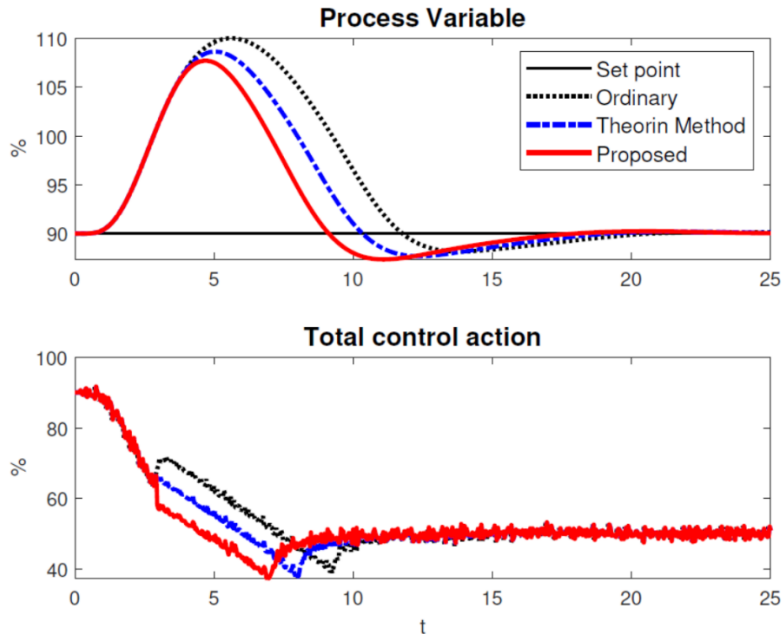


Fig. 3. The process variable (top) and control signal (bottom) of ordinary PID (without any action taken against derivative-backoff problem), Theorin's, and the proposed methods.

### 3. The proposed modification

In this section, a minor modification of the anti-backoff scheme given by Eq. (5) is proposed as follows:

$$\begin{aligned}
 \text{if } y_m = y_{max} \text{ or } y_{min}: \\
 I = I + \alpha D, \\
 D = 0,
 \end{aligned} \tag{6}$$

The rationale behind this modification is that, for severe situations where the process variable is pushed further away from set point, there is a need to amplify or multiply the derivative part by a gain  $\alpha > 1$  before moving it to the integral part and then setting it to zero.

The response of the process given by Eq. (4) using the PID controller employing the proposed anti-derivative backoff scheme given by Eq. (6), with  $\alpha = 2$ , is shown in Fig. 3. As can be seen, the proposed method reduces both the overshoot and settling time compared to ordinary PID and Theorin and Haggglund's method [14]. This is because the amplification of the derivative action results in larger braking effect in the control signal as seen in Fig. 3 (bottom). This improvement in response, however, comes at the expense that the transfer of the control signal at beginning of the process variable saturation, at around time instant of 3 seconds, is not bumpless.

Ordinary PID, Theorin and Haggglund method [14], and the proposed scheme given by Eq. (6) can be also compared in terms of the integral of absolute error (IAE) criterion defined as

$$IAE = \int_0^{\infty} |e(t)| dt. \quad (7)$$

One 100 Monte Carlo simulations are conducted and the average IAE are recorded in Table 1 for disturbance,  $d$ , ranging from 30% to 80%. The proposed scheme outperforms the other two methods achieving the least IAE criterion for disturbances above 40%.

In addition, the proportional, integral, and derivative parts of the control signal for the three methods are shown in Fig. 4. From the Fig. 4 (middle), a larger drop in the integral action is obtained using the proposed modification compared to Theorin and Hagglund method's [14]. This acts as a braking effect reducing the overshoot in process variable. As expected, the proportional and derivative actions in Fig. 4 (top and bottom, respectively) are constant during the interval in which the process variable is above 100%.

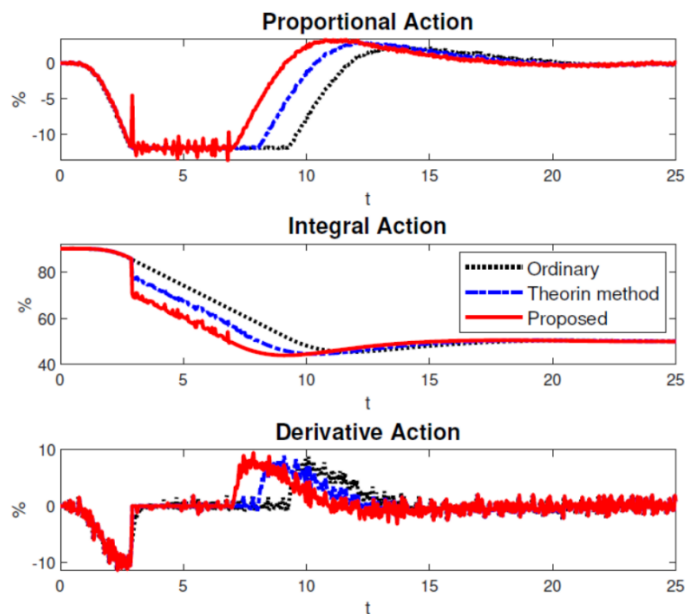


Fig. 4. The proportional (top), integral (middle), and derivative (bottom) control actions of the three methods: Ordinary PID (with no action taken against derivative backoff), Theorin's and the proposed methods.

Table 1. IAE for different values of disturbance  $d$  using ordinary PID, Theorin's method, and proposed method (with fixed  $\alpha = 2$ ) for the process given by Eq. (4).

	$d$ (%)					
Method	30	40	50	60	70	80
Ordinary	79.7	134.8	207.9	299.0	409.2	535.5
Theorin [14]	<b>66.2</b>	95.7	134.5	200.0	257.3	329.3
Proposed	73.3	<b>88.2</b>	<b>105.7</b>	<b>145.0</b>	<b>163.9</b>	<b>205.1</b>

#### 4. Selection of the gain $\alpha$

To determine the optimal value of  $\alpha$ , the system given by the transfer function given by Eq. (4) is simulated with zero measurement noise. The IAE value is plotted against  $\alpha$  for several values of load disturbance as shown in Fig. 5 where it is clear that the optimal  $\alpha$  depends on the amplitude of the disturbance. For example, for a disturbance  $d = 30\%$ , the optimal  $\alpha = 1.5$ , while for a disturbance  $d = 80\%$ ,  $\alpha = 3.0$ , and so on. Note that for  $d = 20\%$ , the amount of disturbance is not enough to cause the process variable to saturate and hence, the backoff problem does not occur in this case. In general, there is a need for larger  $\alpha$  when the size of disturbance is large. Of course, disturbance size would be unknown in practice and there is a need for a reasonable choice for  $\alpha$  that works for most situations.

From Fig. 5, the optimal value of  $\alpha$  ranges approximately from 1.25 to 3.5 for the given set of disturbance. Based on this observation, it is suggested to use fixed  $\alpha = 2$  regardless of the amount of disturbance. This choice is already confirmed in Table 1, where the proposed method, with  $\alpha = 2$ , achieved the best IAE for disturbance  $d \geq 40\%$ . Only for small disturbances, e.g.,  $d = 30\%$ , the method of Theorin and Hagglund [14], which corresponds to  $\alpha = 1$ , outperforms the proposed method.

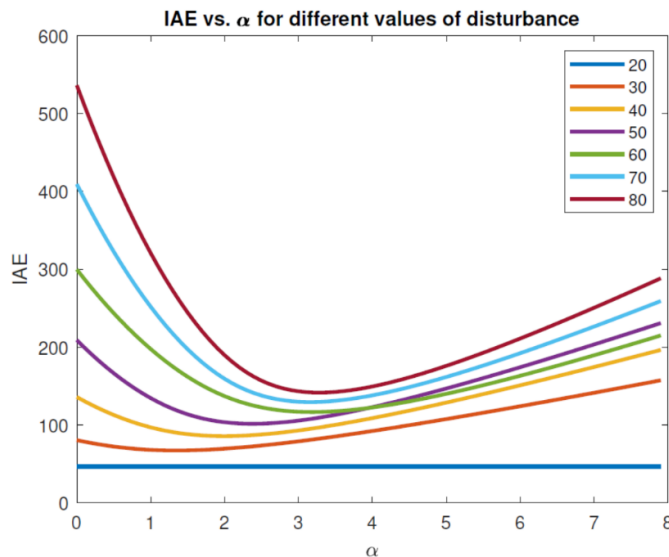


Fig. 5. The IAE vs.  $\alpha$  for different values of step load disturbances ranging from 20% to 80%.

#### 5. Robustness of the proposed method

The empirically suggested value for the gain  $\alpha = 2$  is obtained for the process given by (4). To investigate the robustness of this method, the same PID tuning are used while the true process is modified to

$$P(s) = \frac{1}{(s + 1)^5}. \quad (8)$$

The same experiments are repeated, and the results are given in Table 2. Again, the proposed method outperforms Theorin and Hagglund’s method [14] for all values of disturbances except for low disturbance of 30% as previously obtained for the process given by Eq. (4). This shows that the proposed method is robust against model plant mismatch, i.e., when the process model given by Eq. (4) used for controller tuning is different from the actual plant transfer function given by Eq. (8).

Table 2. IAE for different values of disturbance  $d$  using ordinary PID, Theorin’s method, and proposed method (with fixed  $\alpha = 2$ ) for the process given by Eq. (8).

Method	$d$ (%)					
	30	40	50	60	70	80
Ordinary	114.3	176.7	254.5	348.8	463.5	592.4
Theorin [14]	<b>109.8</b>	145.4	193.1	263.7	348.7	590.1
Proposed	113.4	<b>136.7</b>	<b>171.0</b>	<b>205.4</b>	<b>283.4</b>	<b>587.9</b>

## 6. Conclusions

In this paper, the derivative backoff saturation problem of PID controllers is considered. Although, less attention has been given to this problem, its consequences are similar to the well-known windup problem: large overshoot and settling time in process variable. To solve this problem, a simple method is proposed which significantly reduces overshoot, settling time, and integral of absolute error, and furthermore, is robust against model-plant mismatch. The method involves a gain parameter,  $\alpha$ , whose choice depends on the amount of disturbance affecting the plant. A fixed value  $\alpha = 2.0$  is shown to be reasonable for most sizes of disturbance for the plant under study.

One issue with the proposed anti-backoff method is that it relies on the presence of integral control action [14]. It is interesting to investigate the problem for controllers without integral action.

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