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4 authors, including:

Amgad M Bayoumy Aly
Modern Sciences and Arts University

Ahmed Youssef
Military Technical College

Yehia Z. Elhalwagy
Military Technical College

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Nonlinear Control Law Design For Satellite Fixed Ground Target Tracking

A. M. Elbeltagy[1], A. M. Bayoumy Ali[2], A. M. Youssef [3], Y. Z. Elhalwagy [4]
MTC, Cairo, Egypt

Most of the tracking control algorithms introduced in the available literature care about pointing accuracy and ignore the time elapsed to reach these strict requirements. Actually there is an increasing demand from customers for a fast ground-target tracking even on the expense of pointing accuracy itself. When the target locations and the ground station are both within the satellite's footprint, it could be more important to steer the main imager boresight towards the immediate required target areas before capturing the imaging data. The quick response allows data gathering and downloading in the same communication session for the military intelligence. In this paper, a nonlinear tracking control algorithm is modified and altered to be utilized with exchange momentum actuators (e.g. reaction wheels). The control law uses the commanded attitude rate and acceleration in addition to attitude error and gyroscopic compensation. Tracking error dynamics, equivalent to satellite closed-loop time-varying nonlinear dynamic system, is used alternatively to confirm that a global stability. The proposed controller is applied to fixed ground target tracking task. Generation of the needed target attitude and attitude rate are derived in details. The kinematics of the ground target relative to the satellite is analyzed and presented in the orbit reference frame. In this reference frame, the satellite dynamics are derived from first principles. The body z-axis of satellite is used as a pointing axis in the tracking scheme. Assuming ideality for attitude and orbit determination sensors and symmetric satellite inertia, the validity of proposed controller and target data generator is demonstrated using Matlab/Simulink. Robustness of the proposed control law is discussed against inertia matrix uncertainty. Simulations show that the proposed control law can be used onboard for fast tracking purposes.

Nomenclature

J	=	inertia matrix
ω_B^{ORC}	=	body angular velocity vector
ω_c^{ORC}	=	commanded angular velocity vector
u	=	control torque
$\bar{q}$	=	current attitude quaternion
q_c	=	commanded attitude quaternion
$\delta\bar{q}$	=	quaternion error
D, K	=	derivative and proportioanal gain matrices
ω_n	=	desired average natural frequency
ξ	=	desired damping ratio

[1] PhD student, Aircraft Mechanics Dep., hady_beltagy@yahoo.com
[2] Lecturer, Aircraft Mechanics Dep., dramgad@mep-ls.com
[3] Assoc. Prof, Aircraft Electric system Dep., ammyk_khater@yahoo.com
[4] Assoc. Prof, Guidance and Navigation Dep., elhayehi@isu.edu.

I. Introduction

For several space missions, Earth-pointing satellites are required to point a payload such as a high-gain antenna, camera, and telescope to track a fixed target on the Earth for a certain period of time in order to provide improved long-period up-down-link communication, low-distortion imaging and accurate observation. These missions require ground target tracking control.[1-3] A linear quaternion feedback regulator with open loop decoupling control torque for gyroscopic forces to ensure inertially referenced eigenaxis rotations is proposed in Ref. 3 A tracking controller which deals with much more complicated control task. It considers the target desired rate during realizing the desired target attitude. A tracking controller is realized through a feedback control scheme containing an inner velocity loop that tracks the desired rate command and outer attitude loop that tracks the desired attitude command to achieve tracking via an instantaneous eigenaxis rotation.[1-3] Goeree and Shucker developed a tracking controller with a feedback elastic term for tracking attitude, a feedback viscous term for tracking a desired angular rate, and a feed-forward model-based compensation term.[2] The aim of their algorithm is to enable the telescope mounted along the z-axis of satellite body to track a ground station while passing over and to receive the laser communication signals from the ground station.

A new nonlinear tracking control algorithm based on an attitude error quaternion uses the commanded attitude rate without transformation into the body frame is also introduced.[4] The direct use of the commanded attitude rate simplifies the calculation of its derivative, which is used in the control law. This algorithm is proposed to be used with external actuators for general tracking task. Perturbation dynamics with seven state variables is used to analyze the stability of the system and the tracking performance. Simulation results show that the spacecraft can track the commanded attitude and rate quickly for a non-zero acceleration rate command. This proposed tracking controller was the motivation to propose a modified nonlinear controller for fixed ground target tracking purposes.

Most of the control algorithms introduced previously in the open literature care about pointing accuracy and ignore the time elapsed to reach these strict requirements. Actually there is an increasing demand from customers for a fast ground-target tracking even on the expense of pointing accuracy itself. When the target locations and satellite ground station are both within the satellite's footprint, it may be much more important for the ground station to steer the main imager boresight towards the immediate required target areas before capturing the imaging data. Quick satellite response allows for the military intelligence data gathering and downloading in the same communication session.

In this paper, a nonlinear tracking control algorithm introduced in Ref. 4 is modified and tolerated to be utilized with exchange momentum actuators, e.g. reaction wheels, to track any desired target on the Earth for a nominally Earth- pointing satellite as fast as possible. The proposed tracking controller uses attitude error represented in quaternion, rate error without the need to be represented in satellite body coordinate frame, target acceleration, gyroscopic terms.

The paper is organized as following: Section 2 define the coordinates for attitude transformation through the paper. The satellite nonlinear combined model equipped with three-axis reaction wheels (RW) is presented in section 3. The RW nonlinear attitude tracking controller for fixed ground target is presented in section 4 followed by stability analysis using tracking error dynamics in section 5. The kinematic of fixed ground target tracking including the target desired rate and acceleration is totally covered in details in section 6. A design example with its simulation results using MATLAB /Simulink software is presented in section 7 and finally a conclusion is given in section in section 8.

II. Definition of Coordinate Frames

Several coordinate frames are used coordinates for attitude transformation through the paper as follows.

Orbit Referenced Coordinate frame (ORC) The z-axis points towards the centre of the Earth starting from the satellite in orbit. The y-axis points to the negative orbit normal. The x-axis is chosen to form a right-handed orthogonal reference frame. Therefore, for a circular orbit, the x-axis will be along the velocity vector of the satellite.

Spacecraft Body Fixed Coordinate frame (SBC) The SBC coordinate frame originate from the center of mass of satellite. The x-axis points to the center of the harness side of the satellite. The z-axis is orthogonal to the x-axis. The y-axis is chosen to form a right-handed orthogonal reference system. When the satellite is perfect nadir pointing without any rotation around the z-axis, the SBC and ORC coordinate frames are assumed to be aligned. SBC coordinates originate from the center of mass of satellite. The x-axis points to the center of the harness side of the satellite. The z-axis is orthogonal to the x-axis. The y-axis is chosen to form a right-handed orthogonal reference

system. When the satellite is perfect nadir pointing without any rotation around the z-axis, the SBC and ORC coordinates are assumed to be aligned.
Earth Fixed Coordinate frame (EFC) The EFC frame is Earth-centered. The x-axis will point towards the prime meridian passing through the center of the Royal Greenwich Observatory in London. The z-axis points to the north celestial pole. The y-axis is chosen to form a right-handed orthogonal reference frame.
Earth Centered Inertial Coordinates (EIC) the EIC frame is Earth-centered. The x-axis points towards the mean equinox. The z-axis points to the celestial pole. The y-axis is chosen to form a right-handed orthogonal reference frame.

III. Spacecraft Combined Nonlinear Model

Assuming a rigid satellite equipped with three axis reaction wheels (RW) as internal torque actuators and by recalling the Euler's moment equation, and then the dynamic model of an Earth-pointing satellite is given by[5]

$$J\dot{\omega}_B^I = N_{ext} - \omega_B^I \times (J\omega_B^I + h) - \dot{h} \tag{1}$$

Or,

$$\dot{\omega}_B^I = J^{-1}(N_{ext} - \Omega(\omega_B^I)(J\omega_B^I + h) - \dot{h}) \tag{2}$$

Where J is the spacecraft inertia matrix, $\omega_B^I = (\omega_{B1}^I, \omega_{B2}^I, \omega_{B3}^I)^T$ is the angular velocity vector (ω) in body frame ($_B$) with respect to the inertial reference frame (^{I}), N_{ext} is the external disturbance torque, $h = (h_1, h_2, h_3)^T$ is the RW angular momentum vector, $\dot{h}$ is RW applied torque and $\Omega(\omega_B^I)$ is a skew-symmetric matrix defined by

$$\Omega(\omega_B^I) = \begin{pmatrix} 0 & \omega_{B3}^I & -\omega_{B2}^I \\ -\omega_{B3}^I & 0 & \omega_{B1}^I \\ \omega_{B2}^I & -\omega_{B1}^I & 0 \end{pmatrix} \tag{3}$$

Further assuming the satellite is 3-axis stabilized, and then the absolute angular velocity in the inertial space resolved in the ORC frame is given by [5]

$$\omega_B^I = \omega_B^{ORC} + A_{ORC}^{SBC}\,\omega_O^I \tag{4}$$

Where ω_O^I is the orbital angular rate vector of the satellite motion in ORC frame.
Substitute Eq. (4) into Eq. (2), then the dynamic equation of the Earth-pointing satellite becomes

$$\dot{\omega}_B^{ORC} = J^{-1}(N_{\omega o} + N_{ext} - \Omega(\omega_B^I)(J\omega_B^I + h) - \dot{h}) \tag{5}$$

With

$$\begin{aligned} N_{\omega o} &= -J\,\dot{A}_{ORC}^{SBC}\,\omega_O^I - J\,A_{ORC}^{SBC}\,\dot{\omega}_O^I \\ &= -J\,\Omega(\omega_B^{ORC})A_{ORC}^{SBC}\,\omega_O^I - J\,A_{ORC}^{SBC}\,\dot{\omega}_O^I \end{aligned} \tag{6}$$

Where ω_O^I and $\dot{\omega}_O^I$ are orbit angular rate and orbit angular acceleration represented in ORC frame and derived as follows

$$\omega_O^I = [0 \;\; -\frac{\|r_{sat} \times v_{sat}\|}{\|r_{sat}\|^2} \;\; 0]^T \tag{7}$$

$$\dot{\omega}_O^I = [0 \ \ \frac{-2(r_{sat} \bullet v_{sat})\left\| r_{sat} \times v_{sat} \right\|}{\left\| r_{sat} \right\|^4} \ \ 0]^T \tag{8}$$

An equivalent but more useful form of Eq. (5) for control purposes is given by

$$\begin{aligned} \dot{\omega}_B^{ORC} &= J^{-1}(N_{\omega\infty} + N_{ext} - \Omega(\omega_B^I)(J\omega_B^I) + N_{eff}) \\ \dot{h} &= -\Omega(\omega_B^I)(h) - N_{eff} \end{aligned} \tag{9}$$

Where N_{eff} is the effective torque of the RW and can be simply deduced from the estimated used control law.

The nonlinear attitude kinematics equations of motion of an Earth-pointing satellite can be represented by using various attitude parameters. Representation through quaternion parameter has the property of non-singularity and it is free from the trigonometric component. Therefore, this representation is widely used to study the attitude behaviour of spacecraft. The kinematics of the satellite model is the part which expresses the relation between the attitude and angular velocities of the body and can be described by[6]

$$\begin{aligned} \dot{q} &= -\frac{1}{2}\Omega(\omega_B^{ORC})q + \frac{1}{2}q_4\omega_B^{ORC} \\ \dot{q}_4 &= -\frac{1}{2}(\omega_B^{ORC})^T q \end{aligned} \tag{10}$$

The satellite attitude w.r.t ORC frame as a reference frame is determined by its quaternion $\bar{q}$. Both q and q_4 of the quaternion $\bar{q}$ are defined as

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} e_1 \sin\left(\frac{\varphi}{2}\right) \\ e_2 \sin\left(\frac{\varphi}{2}\right) \\ e_3 \sin\left(\frac{\varphi}{2}\right) \end{bmatrix} \tag{11}$$

$$q_4 = \cos\left(\frac{\varphi}{2}\right) \tag{12}$$

$$\bar{q} = \begin{bmatrix} q \\ q_4 \end{bmatrix} \tag{13}$$

In Eq. (11), e_1, e_2 and e_3 are the components of the rotation axis unit vector along the reference frame; φ is the rotation angle.

The combined dynamic and kinematic, Eq. (5) and Eq. (10), give the general nonlinear model for the spacecraft angular motion with ten state variables

$$\frac{d}{dt}\begin{bmatrix}\omega_B^{ORC}\\ q\\ q_4\\ h\end{bmatrix}=\begin{bmatrix}J^{-1}(N_{\omega o}+N_{ext}-\Omega(\omega_B^I)(J\omega_B^I)+N_{eff})\\ -\frac{1}{2}\Omega(\omega_B^{ORC})q+\frac{1}{2}q_4\omega_B^{ORC}\\ -\frac{1}{2}(\omega_B^{ORC})^T q\\ -\Omega(\omega_B^I)(h)-N_{eff}\end{bmatrix} \tag{14}$$

In most spacecraft (SC) guidance and control (G&C) systems, the desired reference is generated by the guidance subsystem Fig. 1.

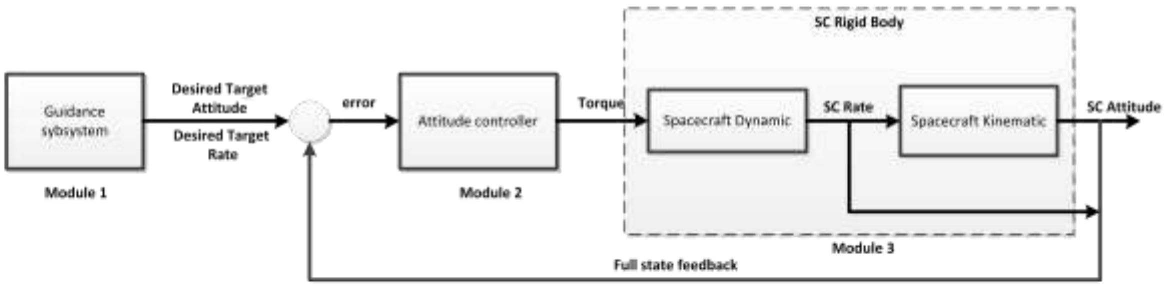

Figure 1. Overall system G&C block diagram used in simulation[9]

IV. RW nonlinear attitude tracking controller

Based on the nonlinear tracking control law introduced in Ref. 4 which use attitude error quaternion and desired rate without transformation in the body frame, a modified proposed RW nonlinear tracking controller for ground target tracking purposes is written as follows:

$$\dot{h}=N_{\omega o}+N_{ext}-\Omega(\omega_B^I)(J\omega_B^I+h)-J\dot{\omega}_c+D(\omega_B^{ORC}-\omega_c^{ORC})+K\,\delta q \tag{15}$$

Where δq is attitude error quaternion vector of attitude error quaternion $\delta\bar{q}$, which is defined as follows

$$\delta\bar{q}\equiv\begin{bmatrix}\delta q\\ \delta q_4\end{bmatrix}\equiv(\bar{q}_c^{-1}.\ \bar{q}) \tag{16}$$

$$\delta\bar{q}\equiv\begin{bmatrix}q_{4c} & q_{3c} & -q_{2c} & -q_{1c}\\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c}\\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c}\\ q_{1c} & q_{2c} & q_{3c} & q_{4c}\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} \tag{17}$$

Where $\bar{q}_c^{-1}$ is the inverse of $\bar{q}_c$, $(\bar{q}_c^{-1}.\ \bar{q})$ is quaternion multiplication. Knowing that $\delta\bar{q}$ is still a quaternion which has a physical meaning as it represents the attitude difference between $\bar{q}$ and $\bar{q}_c$.

In such case effective wheel torque is

$$N_{eff}=N_{\omega o}+N_{ext}-\Omega(\omega_B^I)J\,\omega_B^I-J\,\dot{\omega}_c^{ORC}+D(\omega_B^{ORC}-\omega_c^{ORC})+K\,\delta q \tag{18}$$

Substitute Eq. (15) into Eq. (5) gives

$$\dot{\omega}_B^{ORC} - \dot{\omega}_c^{ORC} = J^{-1}(-D\,(\omega_B^{ORC} - \omega_c^{ORC}) - K\,\delta q\,) \tag{19}$$

Eq. (19) in addition to Eq. (10) form the closed-loop time-varying nonlinear dynamic system of the spacecraft attitude with the control law (15).

V. Stability analysis

A. Closed-loop solution

It is clear that the following is a solution of equation (19) and equation (10)

$$\begin{aligned} \omega_B^O &= \omega_c^{ORC} \\ q &= q_c \end{aligned} \tag{20}$$

Solution (20) means theoretically that it is possible for the spacecraft to follow the commanded attitude and attitude rate without restriction although in reality there are always certain limits in using different actuators. The terms $N_{\omega o}$, $\Omega(\omega_B^I)(\,J\,\omega_B^I + h)$ and $J\,\dot{\omega}_c^{ORC}$ are essential to be included in the proposed RW controller dedicated for tracking task of nominal earth pointing satellite. Including these terms ensure that the solution (20) always exist and hence can be applied in different maneuver conditions (e.g. rest to rest maneuver, tracking commanded attitude with constant rate). The stability analysis can draw a clear conclusion concerning the ability to track the commanded attitude when solution (20) exists.

The commanded rate and its derivative ω_c^{ORC} and $\dot{\omega}_c^{ORC}$ respectively will estimated later for fixed ground target with aid of GPS data. Therefore, the nonlinear controller is applicable for spacecraft attitude control and command tracking.

B. Tracking error dynamics

Generally asymptotic stability analysis is required to ensure that the satellite can follow the commanded attitude with commanded attitude rate from any condition of both ω_B^{ORC} and q. The stability analysis is started by studying the local asymptotic stability, at which the initial values of both ω_B^{ORC} and q are near the solution (20). Once this local stability is confirmed, a global asymptotic stability, at which any values of both ω_B^{ORC} and q are assumed, is further searched. The main objective of this paper is to find the constant gain matrices D and K having the globally asymptotic stable solution (20) and guaranteeing the needed solution converges performance. The tracking error dynamics introduced in Ref. 4 are employed for the indirect stability analysis alternatively to the non-convenient direct analysis of solution (20). The tracking error dynamics defined in the following are always valid whatever the differences between the commanded rates and the actual rates are large or small.

We define the following state variable x to represent the tracking error dynamics as follows

$$x \equiv \begin{pmatrix} x_{13} \\ x_{46} \\ x_7 \end{pmatrix} \equiv \left(x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \right)^T \tag{21}$$

Where,

$$
\begin{aligned}
x_1 &= \omega_{1B}^{ORC} - \omega_{1c}^{ORC} \\
x_2 &= \omega_{2B}^{ORC} - \omega_{2c}^{ORC} \\
x_3 &= \omega_{2B}^{ORC} - \omega_{3c}^{ORC} \\
x_4 &= \Delta q_1 \\
x_5 &= \Delta q_2 \\
x_6 &= \Delta q_3 \\
x_7 &= \Delta q_4
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
x_{13} &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T \\
x_{46} &= \begin{pmatrix} x_4 & x_5 & x_6 \end{pmatrix}^T
\end{aligned} \tag{23}
$$

With,

$$
\Delta\bar{q} \equiv \bar{q} - \bar{q}_c \equiv \begin{pmatrix} \Delta q \\ \Delta q_4 \end{pmatrix} \equiv \begin{pmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \end{pmatrix} \equiv \begin{pmatrix} q_1 - q_{1c} \\ q_2 - q_{2c} \\ q_3 - q_{3c} \\ q_4 - q_{4c} \end{pmatrix} \tag{24}
$$

Substituting (21-24) into Eq. (19) and Eq. (10), getting similar formula deduced in Ref. 4 for the tracking error dynamics with minor changes as

$$
\begin{aligned}
\dot{x}_{13} &= f_{13} = -\bar{D}\, x_{13} - \bar{K}\,[\bar{q}_c]\begin{pmatrix} x_{46} \\ x_7 \end{pmatrix} \\
\dot{x}_{46} &= f_{46} = \frac{1}{2} Q_c\, x_{13} + \frac{1}{2} q_{4c}\, x_{13} - \frac{1}{2}\Omega(\omega_c^{ORC})\, x_{46} + \frac{1}{2}\omega_c\, x_7 - \frac{1}{2} X x_{46} + \frac{1}{2} x_7\, x_{13} \\
\dot{x}_7 &= f_7 = -\frac{1}{2}(q_c^T\, x_{13} + \omega_c^{ORCT}\, x_{46} + x_{13}{}^T x_{46})
\end{aligned} \tag{25}
$$

Where ,

$$
\bar{D} = J^{-1} D \tag{26}
$$

$$
\bar{K} = J^{-1} K \tag{27}
$$

$$
[\bar{q}_c] \equiv \begin{pmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \end{pmatrix} \tag{28}
$$

$$
Q_c \equiv \begin{pmatrix} 0 & -q_{3c} & q_{2c} \\ q_{3c} & 0 & -q_{1c} \\ -q_{2c} & q_{1c} & 0 \end{pmatrix} \tag{29}
$$

$$X \equiv \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} \tag{30}$$

The tracking error dynamics Eq. (25) is equivalent to Eq. (19) and Eq. (10). This equation (Eq. (25)) has an equilibrium point $x = 0$, which corresponds to solution (20). Therefore, instead of studying the deviation of ω_B^{ORC} and q from ω_c^{ORC} and q_c for Eq. (19) and Eq. (10), we may simply study the stability of the tracking error Eq. (25) with respect to the equilibrium point x = 0.

C. Global stability analysis

A Lyapunov function based global stability analysis of the time-varying nonlinear tracking error dynamics Eq. (25) is employed. The states variable x in this error dynamics represents the deviation from orbit referenced satellite body rate, quaternion and desired target rate and quaternion respectively. Although the states variable x of the nonlinear dynamic system Eq.(19) and Eq. (10) with the proposed control law (15) is different from those used in Ref. 5 , similarity in the structure of tracking error dynamics is enough to use theorem 2. Conditions In Ref. 4 to guarantee stability. It is proven that when $\bar{K}^{-1}$ is symmetric and positive definite and that $\bar{K}^{-1}\bar{D}$ is positive definite, the equilibrium point $x = 0$ of tracking error dynamics is globally stable.

The candidate Lyapunov function V , which is independent of time and is radially unbound , is defined by

$$V(x) = x^T \bar{P} x \tag{31}$$

Where ,

$$\bar{P} = \begin{pmatrix} P_{3\times3} & 0_{3\times4} \\ 0_{4\times3} & I_{4\times4} \end{pmatrix} \tag{32}$$

P is a symmetric and positive definite matrix. $I_{4\times4}$ is an identity matrix of order 4. The total time derivative of V along the trajectories of the tracking error dynamics is proved to be negative semidefinite when P is selected

$$P = \frac{1}{2}\bar{K}^{-1} \tag{33}$$

Then

$$\dot{V}(x) = -2x_{13} P \bar{D} x_{13} \tag{34}$$

Therefore the equilibrium point $x = 0$ of Eq.(25) is globally stable. Furthermore using theorem 8.4 of Ref. 7 , we conclude that x_{13} approaches zero as $t \to \infty$.

Assuming $\bar{D}$ and $\bar{K}$ are diagonal matrices expressed as

$$\begin{aligned} \bar{D} &= diag(d_1, d_2, d_3) \\ \bar{K} &= diag(\bar{k}_1, \bar{k}_2, \bar{k}_3) \end{aligned} \tag{35}$$

The dynamic behaviour of the nonlinear system Eq.(25) can be controlled around the equilibrium point in the direction of eigenvalues with nonzero real parts of the Jacobian matrix.[4]

The characteristic's equation of the Jacobian matrix for the special case when $\omega_c^{ORC} = 0$ can be expanded as [4]

$$\lambda(\lambda^2 + \bar{d}_1\lambda + \frac{\bar{k}_1}{2})(\lambda^2 + \bar{d}_2\lambda + \frac{\bar{k}_2}{2})(\lambda^2 + \bar{d}_3\lambda + \frac{\bar{k}_3}{2}) = 0 \tag{36}$$

Expressing Eq. (36) by the desired damping ratio ξ and natural frequency ω_n, then the gains matrices are related directly to the desired damping ratio ξ and natural frequency ω_n as [4]

$$\begin{aligned} \bar{d}_1 &= 2\xi_1\omega_{n1} & \bar{k}_1 &= 2\omega_{n1}^2 \\ \bar{d}_2 &= 2\xi_2\omega_{n2} & \bar{k}_2 &= 2\omega_{n2}^2 \\ \bar{d}_3 &= 2\xi_3\omega_{n3} & \bar{k}_3 &= 2\omega_{n3}^2 \end{aligned} \tag{37}$$

Since actuator constraints are considered in this paper, these gain matrices figured using Eq. (37) can be used as initial values guide. Matlab/ Simulink Optimization tools are used to adjust the dynamic behaviour of the nonlinear system Eq. (25) around the equilibrium point. Tracking error is the selected parameter to be optimized within short time.

VI. Kinematic of fixed ground target tracking

Target Tracking Mode will most likely be activated from Nominal Mode as soon as a predefined ground target location (eg. Ground station) is in range. Knowledge of the satellite position is therefore required by the on-board computer. The method that will be used to calculate the desired attitude and angular rates for tracking a target was presented by Chen et al. in 2000.[8]

The satellite will need to be able to determine whether the target is in range (i.e. in the satellite's field of view of the Earth). The distance D_{max} from the satellite to the furthest point on Earth that the satellite can "see" can be calculated as

$$D_{max} = \sqrt{(R_E + h)^2 - R_E} \tag{38}$$

Where R_E is the radius of the earth and h is satellite altitude.

An illustration of the above-mentioned geometry can be seen in Fig. 2.

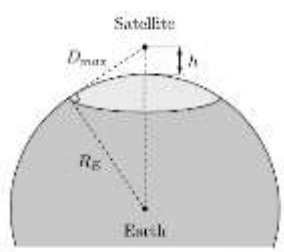

Figure 2. Geometry of maximum target tracking distance

Comparing the current distance to the target with maximum target tracking distance allow the on-board control algorithms to know if the target is within the visibility zone or not and hence tracking task can be started.

Since the nominal mode of earth orbiting satellite usually maintains the satellite body frame aligned with orbital (ORC) frame, it is preferable to describe the kinematics of the ground target with respect to the in-orbit satellite in ORC frame. The vector from the satellite to the ground target represented in ORC will specify the tracking direction of the selected pointing axis which may be the positive mounting axis of a camera or antenna.

Firstly the vector from the centre of the Earth to the ground target with respect to EFC frame from the given geocentric latitude φ (assuming a spherical Earth) and longitude λ will be derived as

$$x_T^{EFC} = R_E \begin{bmatrix} \cos(\lambda)\cos(\varphi) \\ \sin(\lambda)\cos(\varphi) \\ \sin(\varphi) \end{bmatrix} \quad (39)$$

Using the transformation matrix from EFC frame to EIC frame, A_{EFC}^{EIC}, the location of the target with respect to the EIC frame is given by

$$x_T^{EIC} = A_{EFC}^{EIC} x_T^{EFC} \quad (40)$$

Assuming that the Earth has a constant angular rate ω_E around its rotation axis during the tracking maneuver, then the attitude transformation matrix A_{EFC}^{EIC} from the EFC frame to EIC frame can be represented by

$$A_{EFC}^{EIC} = \begin{bmatrix} \cos(\omega_E t + \alpha) & -\sin(\omega_E t + \alpha) & 0 \\ \sin(\omega_E t + \alpha) & \cos(\omega_E t + \alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (41)$$

Where t is time scalar and α is the initial phase between the x-axes of both EFC frame and EIC frame. x_T^{EIC} and r_{sat} (the satellite's position vector in EIC frame , as obtained by onboard GPS receiver or produced by any orbit propagator for simulation purposes) can then be used to calculate $x_{S/T}^{ORC}$, the vector from the satellite to the target represented in orbit frame, thus

$$x_{S/T}^{ORC} = A_{EIC}^{ORC}(x_T^{EIC} - r_{sat}) \quad (42)$$

as A_{EIC}^{ORC}, is the transformation matrix from EIC frame to ORC frame and defined by

$$A_{EIC}^{ORC} = [\hat{u}\ \hat{v}\ \hat{w}]^T \quad (43)$$

Where,

$$\hat{w} = -\frac{r_{sat}}{\|r_{sat}\|}$$
$$\hat{v} = -\frac{r_{sat} \times v_{sat}}{\|r_{sat} \times v_{sat}\|} \quad (44)$$
$$\hat{u} = \hat{v} \times \hat{w}$$

If the magnitude of $x_{S/T}^{ORC}$ is larger than D_{max}, then the target is not in range and the satellite will continue operating in nominal mode. However, if $\|x_{S/T}^{ORC}\| < D_{max}$, target tracking mode will be activated. |actually the direction of $x_{S/T}^{ORC}$ is of use to the satellite's ADCS, hence the unit vector $u_{S/T}^{ORC}$ must be calculated by normalised$x_{S/T}^{ORC}$, thus

$$u_{S/T}^{ORC} = \frac{x_{S/T}^{ORC}}{\|x_{S/T}^{ORC}\|} \quad (45)$$

Target tracking task requires that the mounting axis of the commissioned payload being controlled to point towards the direction of $u_{S/T}^{ORC}$. In most cases, the commissioned payload, u_{com}^{SBC}, such as a camera, antenna or telescope is usually mounted along the z-axis of the satellite body. This configuration theoretically allows the ADCS designers to achieve the task with minimum resources (i.e. task can be achieved by only using the x- and y- reaction wheels). Nevertheless this configuration permits specifically controlling the angle around the mounting axis. Moreover, defining $A_{ORC\_d}^{SBC}$ to represent the desired attitude from ORC frame to SBC frame, then

$$A_{ORC\_d}^{SBC}\, u_{S/T}^{ORC} = u_{com}^{SBC} \tag{46}$$

Let the 1-2-3 sequence of rotations is used for the description of the desired attitude matrix $A_{ORC\_d}^{SBC}$ for tracking. Substituting such attitude matrix[9] in terms of $\emptyset, \theta$ and ψ respectively represent the roll, pitch and yaw angles in Eq. (46) leads to

$$u_{S/T}^{ORC} = [\sin\theta \quad -\cos\theta\sin\phi \quad \cos\theta\cos\phi]^T \tag{47}$$

Based upon Eq. (47) the roll and pitch angles ϕ and θ for tracking can be easily solved. Assuming a constant yaw ψ is required during tracking, the desired attitude matrix $A_{ORC\_d}^{SBC}$ can be computed. Therefore, the referenced quaternion command q_c can be derived. Accordingly, δq can be solved using Eq. (17).

The satellite's angular rate for target tracking will not be zero, since the target is fixed with respect to the Earth and will thus be rotating relative to the orbiting satellite. The commanded angular rate ω_c^{ORC} can be calculated as represented in[8]

$$\omega_c^{ORC} = u_{S/T}^{ORC} \times \dot{u}_{S/T}^{ORC} \tag{48}$$

Due to the assumption of a constant yaw angle during tracking, the desired angular rate along the z-axis will be zero. Based upon Eqs. (48) , ω_c^{ORC} can be reduced to [8]

$$\omega_c^{ORC} = \begin{bmatrix} -\dfrac{\dot{u}_{S/T\_y}^{ORC}}{u_{S/T\_z}^{ORC}} & \dfrac{\dot{u}_{S/T\_x}^{ORC}}{u_{S/T\_z}^{ORC}} & 0 \end{bmatrix}^T \tag{49}$$

The commanded angular rate $\dot{\omega}_c^{ORC}$ can be derived by tacking the time derivative of Eq. (48)

$$\begin{aligned} \dot{\omega}_c^{ORC} &= \dot{u}_{S/T}^{ORC} \times \dot{u}_{S/T}^{ORC} + u_{S/T}^{ORC} \times \ddot{u}_{S/T}^{ORC} \\ &= u_{S/T}^{ORC} \times \ddot{u}_{S/T}^{ORC} \end{aligned} \tag{50}$$

The $\ddot{u}_{S/T}^{ORC}$ term can be derived by first tacking the time derivative of Eq. (45) yields

$$\dot{u}_{S/T}^{ORC} = \frac{1}{\left\| x_{S/T}^{ORC} \right\|}[I_3 - u_{S/T}^{ORC}(u_{S/T}^{ORC})^T]\dot{x}_{S/T}^{ORC} \tag{51}$$

So $\ddot{u}_{S/T}^{ORC}$ can be derived by tacking the time derivative of Eq. (51) yields

$$\ddot{u}_{S/T}^{ORC} = \frac{\ddot{x}_{S/T}^{ORC}}{\left\| x_{S/T}^{ORC} \right\|} - \frac{1}{\left\| x_{S/T}^{ORC} \right\|^2}\frac{d(\left\| x_{S/T}^{ORC} \right\|)}{dt}\dot{x}_{S/T}^{ORC} - \frac{d}{dt}\left(\frac{u_{S/T}^{ORC}(u_{S/T}^{ORC})^T\dot{x}_{S/T}^{ORC}}{\left\| x_{S/T}^{ORC} \right\|}\right) \tag{52}$$

But generally for any vector V ,

$$\frac{d}{dt}\left(\left\|V\right\|\right)=\frac{1}{\left\|V\right\|}(V)^{T}\dot{V} \tag{53}$$

Using Eq. (53) and substitute the corresponding relations in Eq. (54) and make some manipulation yields

$$\begin{aligned}\dot{u}_{S/T}^{ORC} = &\frac{\ddot{x}_{S/T}^{ORC}}{\left\|x_{S/T}^{ORC}\right\|}-\Big(\frac{1}{\left\|x_{S/T}^{ORC}\right\|^{3}}(x_{S/T}^{ORC})^{T}\dot{x}_{S/T}^{ORC}\Big)\dot{x}_{S/T}^{ORC}+\frac{1}{\left\|x_{S/T}^{ORC}\right\|^{3}}(x_{S/T}^{ORC})^{T}\dot{x}_{S/T}^{ORC}\left(u_{S/T}^{ORC}(u_{S/T}^{ORC})^{T}\dot{x}_{S/T}^{ORC}\right)\\ &-\frac{1}{\left\|x_{S/T}^{ORC}\right\|}\left(\dot{u}_{S/T}^{ORC}(u_{S/T}^{ORC})^{T}\dot{x}_{S/T}^{ORC}+u_{S/T}^{ORC}(\dot{u}_{S/T}^{ORC}\dot{x}_{S/T}^{ORC}+u_{S/T}^{ORC}\ddot{x}_{S/T}^{ORC})\right)\end{aligned} \tag{54}$$

Furthermore from Eq. (42) , $\dot{x}_{S/T}^{ORC}$ can be determined as

$$\dot{x}_{S/T}^{ORC}=\dot{A}_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-r_{sat})+A_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-v_{sat}) \tag{55}$$

Tacking the derivative of Eq. (55) get

$$\ddot{x}_{S/T}^{ORC}=\frac{d}{dt}(\dot{A}_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-r_{sat}))+\frac{d}{dt}(A_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-v_{sat})) \tag{56}$$

As,

$$\frac{d}{dt}(\dot{A}_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-r_{sat}))=\ddot{A}_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-r_{sat})+\dot{A}_{EIC}^{ORC}(\dot{A}_{EFC}^{EIC}x_{T}^{EIC}-A_{EFC}^{EIC}\dot{x}_{T}^{EIC}) \tag{57}$$

$$\frac{d}{dt}(A_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-v_{sat})=\dot{A}_{EIC}^{ORC}(A_{EFC}^{EIC}x_{T}^{EIC}-v_{sat})+A_{EIC}^{ORC}(\dot{A}_{EFC}^{EIC}x_{T}^{EIC}-a_{sat}) \tag{58}$$

Where,

$$\begin{aligned}\dot{A}_{EIC}^{ORC}&=[\dot{\hat{u}}\ \dot{\hat{v}}\ \dot{\hat{w}}]^{T}\\ \dot{\hat{w}}&=-\frac{1}{\left\|r_{sat}\right\|}(I_{3}-\hat{w}\hat{w}^{T})v_{sat}\\ \dot{\hat{v}}&=0\\ \dot{\hat{u}}&=\frac{1}{\left\|r_{sat}\right\|}\hat{v}\times[(I_{3}-\hat{w}\hat{w}^{T})v_{sat}]\\ \dot{A}_{EFC}^{EIC}&=\omega_{E}\begin{bmatrix}-\sin(\omega_{E}t+\alpha) & -\cos(\omega_{E}t+\alpha) & 0\\ \cos(\omega_{E}t+\alpha) & -\sin(\omega_{E}t+\alpha) & 0\\ 0 & 0 & 0\end{bmatrix}\\ \ddot{A}_{EIC}^{ORC}&=[\ddot{\hat{u}}\ \ddot{\hat{v}}\ \ddot{\hat{w}}]^{T}\end{aligned} \tag{59}$$

Differentiating $\dot{\hat{w}}$, $\dot{\hat{v}}$ and $\dot{\hat{u}}$ in Eq. (61) and using Eq. (53) to get the column vectors of $\ddot{A}_{EIC}^{ORC}$ as follows

$$\dot{\hat{w}} = -\frac{a_{sat}}{\|r_{sat}\|} + \frac{1}{\|r_{sat}\|^2}\left(\frac{1}{\|r_{sat}\|} r_{sat}{}^T v_{sat}\right) v_{sat} + \frac{\dot{\hat{w}}\hat{w}^T}{\|r_{sat}\|} v_{sat}$$
$$+\left(\frac{\dot{\hat{w}} v_{sat} + \hat{w}^T a_{sat}}{\|r_{sat}\|} - \frac{1}{\|r_{sat}\|^2}\left(\frac{1}{\|r_{sat}\|} r_{sat}{}^T v_{sat}\right)\hat{w}^T v_{sat}\right)\hat{w} \tag{60}$$

$$\dot{\hat{v}} = 0 \tag{61}$$

$$\dot{\hat{u}} = -\frac{1}{\|r_{sat}\|^3} r_{sat}{}^T v_{sat}\, \hat{v} \times v_{sat} + \frac{1}{\|r_{sat}\|}(\hat{v} \times a_{sat}) + \frac{1}{\|r_{sat}\|^3} r_{sat}{}^T v_{sat}\, (\hat{v} \times \hat{w}\hat{w}^T v_{sat})$$
$$- \frac{1}{\|r_{sat}\|}(\hat{v}(\dot{\hat{w}}\hat{w}^T v_{sat} + \hat{w}\hat{w}^T a_{sat})) \tag{62}$$

Where a_{sat} is satellite acceleration vector as [5]

$$a_{sat} = \ddot{r}_{sat} = -\frac{\mu}{\|r_{sat}\|^3} r_{sat} \tag{63}$$

So finally using Eqs. (53-61) in addition to Eq. (63) to get $\ddot{u}_{S/T}^{ORC}$ in Eq. (62) and substitute in Eq. (52) to obtain $\dot{\omega}_c^{ORC}$

A Matlab/Simulink software code has been built to simulate the satellite fixed ground target tracking task. The overall system simulator includes many subsystems or modules. The main layer of the paper's built simulator [9] is shown in Fig. 3.

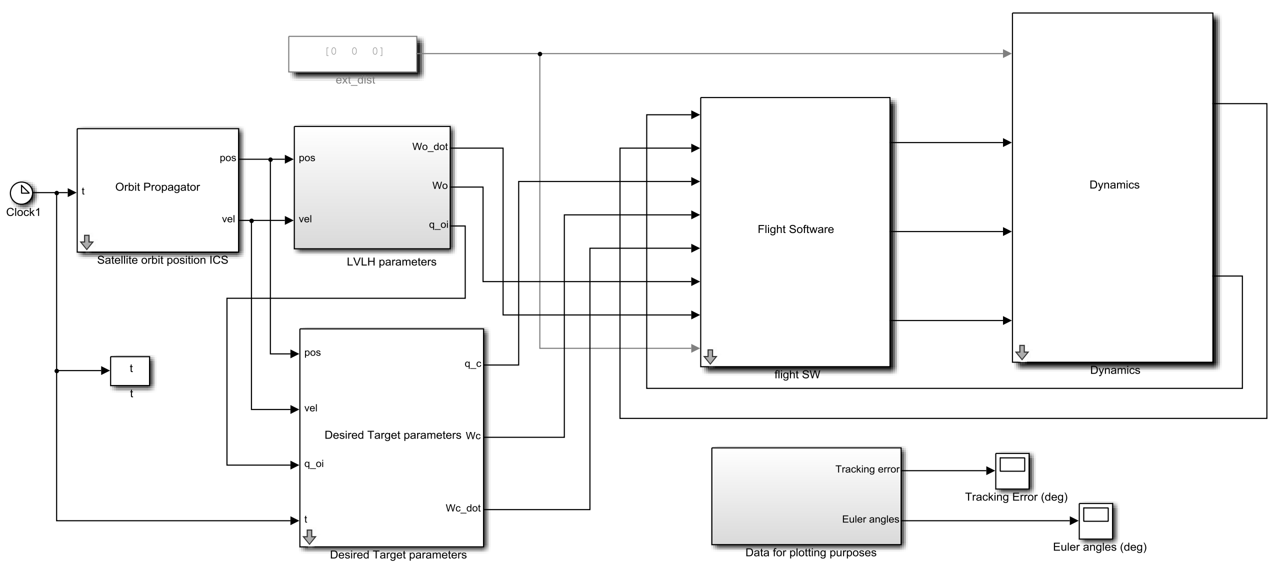

Figure 3. Main layer of satellite fixed ground Target tracking task[9]

VII. Design example

In this section, a typical design example has been used in simulation tests to verify the performance of the modified proposed tracking controller presented above. An imaginary satellite in a low-Earth-eccentric orbit is used as an example[10] during these simulations. In order to investigate the attitude change of the satellite, we define the roll, pitch and yaw angles respectively to represent the rotations of the satellite body x, y and z axes with respect to the ORC coordinates. The simulation parameters are given in Table 1. During simulation, we assume perfect attitude knowledge. We also assume perfect measurements of the vectors r_{sat} and v_{sat} from a GPS receiver.
Matlab/ Simulink Optimization tools is used to search almost best gains to allow the tracking process meets the required rapid pointing to the target with a little relaxed accuracy. This yields the following gains

$$K = \begin{bmatrix} 21.3195 & 0 & 0 \\ 0 & 17.7705 & 0 \\ 0 & 0 & 13.2615 \end{bmatrix} \quad and \quad D = \begin{bmatrix} 18.6578 & 0 & 0 \\ 0 & 18.6599 & 0 \\ 0 & 0 & 17.4118 \end{bmatrix}$$

Simulation is made for one of the overpass flight. Fig. 4-5 shows that the pointing error of the proposed tracking controller of Eq. (15) succeeded to maneuver satellite through a large angle to a predetermined attitude with the required relaxed accuracy 0.32^o within only $40\,sec$ which is well below the requirements. Fig. 6-7 show the quaternion error and rate error simulation results. It is seen that both orbit referenced satellite attitude and desired attitude are almost coincide within only 30 sec.

Table 1. Simulation parameters

Required Tracking accuracy	0.35^o within 1 min of being in satellite FOV
Target position	$x_T^{EIC} = [4021.9\ \ -35.1\ \ 4933.6]^T\ km$
Unit vector of commissioned axis in SBC frame	$u_{com}^{SBC} = \ [0\ \ 0\ \ 1]^T$
Moment of inertia	$J = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 32 \end{bmatrix}\ kgm^2$
Reaction wheel	Max torque $Tq = 0.02\ Nm$, Max momentum $h_w = 4Nms$ Initial Momentum = $[0.4\ \ -0.5\ \ 0.6]^T\ Nms$
Orbit parameters	Perigee altitude = 650 km , Inclination $i = 64.5^o$ Eccentricity e = 0.3, argument of perigee $\omega = 0^o$ Right ascension of ascending node $\Omega = 10^o$ Initial mean anomaly $M_0 = 0^o$
Initial phase	$\alpha = 0^0$
Sample time	1 sec

The orbit referenced angular velocity ω_B^{ORC} of the satellite body does track the desired angular rate command ω_c with acceptable limits during the tracking period at which the target is within the FOV of the satellite.

The activities of the three-axis reaction wheels represented in wheel realized torque and wheel momenta are shown in Figs. 8-9. During the tracking period, the torques of three axis reaction wheels is far from saturation. The reaction wheels will apply maximum torque ($0.02\ Nm.$) to provide a bang-bang control manoeuvres to meet the required target tracking within specified limited time with relaxed pointing accuracy. The wheel momentum is kept well away below the permissible limits ($4\ Nms.$). This safe pattern can guarantee allowable gradually momentum build up caused by secular external disturbance torques such as the gravity gradient, aerodynamic and solar pressure forces which are not considered in simulation in this paper.

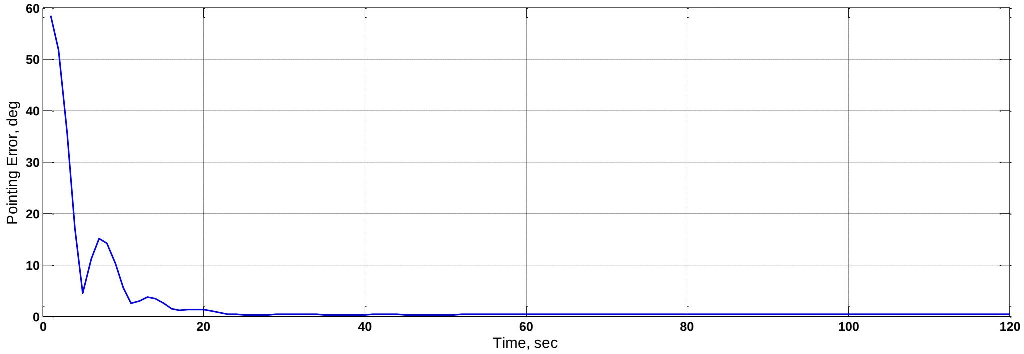

Figure 4. Ground Traget Tracking Error

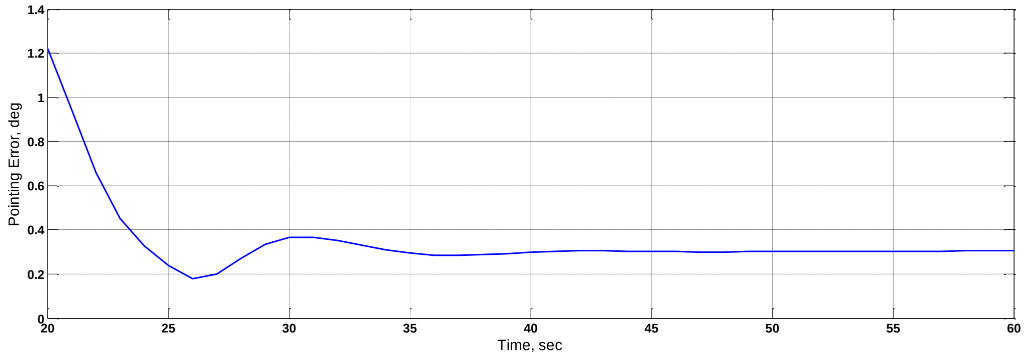

Figure 5. Ground Traget Tracking Error (Zoom in)

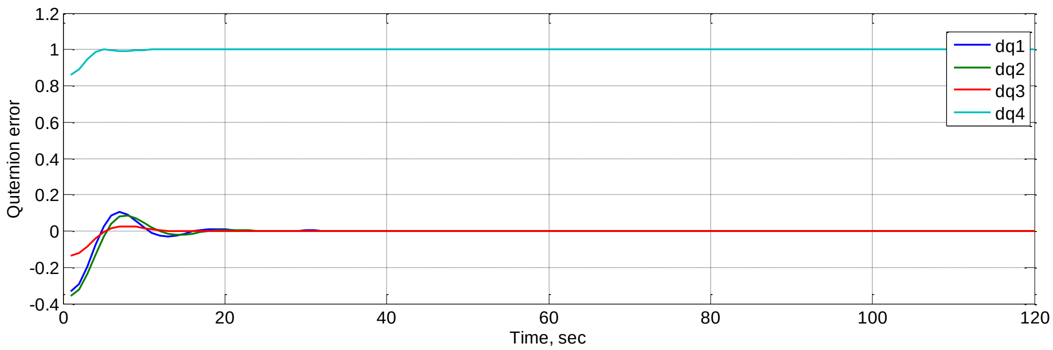

Figure 6. Quaternion error

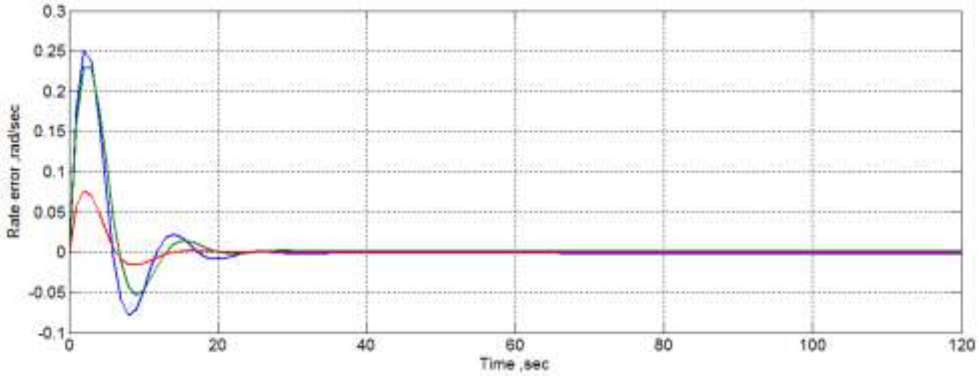

Figure 7. Rate difference error

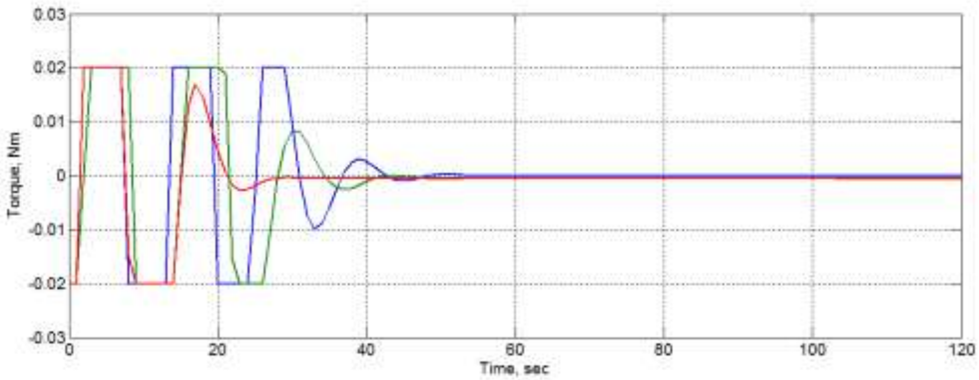

Figure 8. Wheel torque

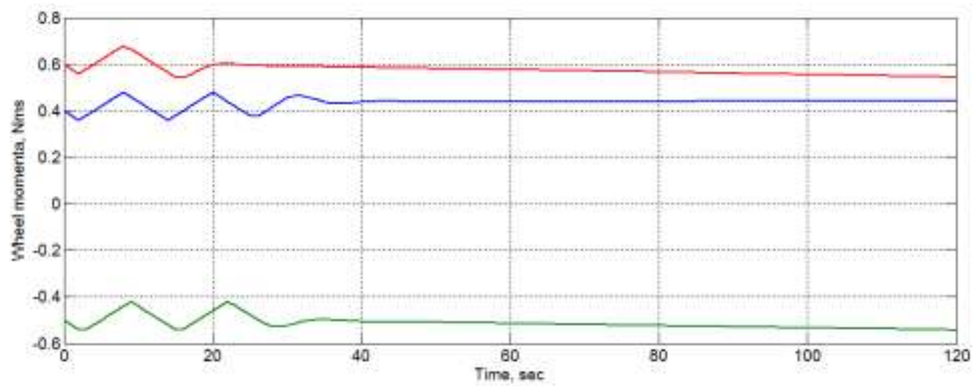

Figure 9. Wheel momenta

Due to the feedback nature of proposed controller, this controller is robust against model uncertainties. Simulations were also done to investigate the robust behaviour of the tracking controller against a ±20% error in the moment of inertia tensor of the satellite. These simulations show in Fig. 10 that the tracking error can still be maintained within required limits during the fast ground target tracking.

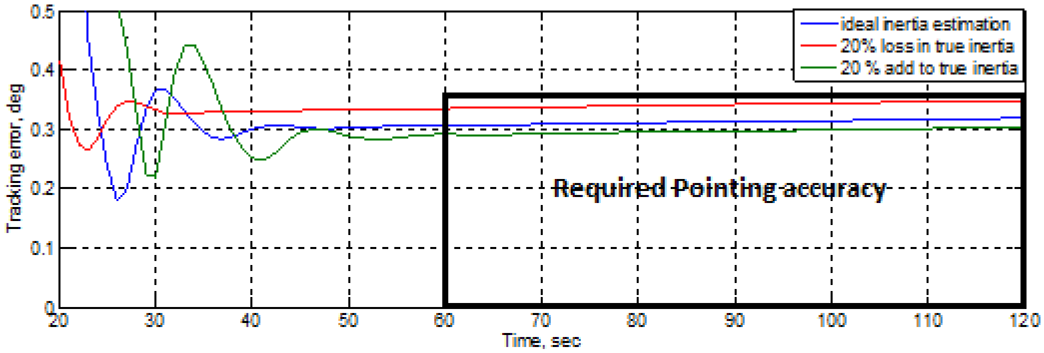

Figure 10. Ground Traget Tracking Error with up to ±20% inertia uncertainty (Zoom in)

The gain selection of the proposed RW nonlinear controller can also be selected to guarantee for much more better tracking accuracy. Matlab/ Simulink Optimization tools is used again to search for almost best gains to allow the tracking process meets better tracking accuracy. This yields the following gains

$$K = \begin{bmatrix} 23.9367 & -11.8354 & -3.4290 \\ -0.8835 & 15.0608 & -9.3174 \\ 2.8083 & 0.7925 & 17.0515 \end{bmatrix} \quad and \quad D = \begin{bmatrix} 14.1279 & 19.1352 & 8.3819 \\ -6.5216 & 20.7349 & 12.1606 \\ -0.6191 & -5.8848 & 19.9367 \end{bmatrix}$$

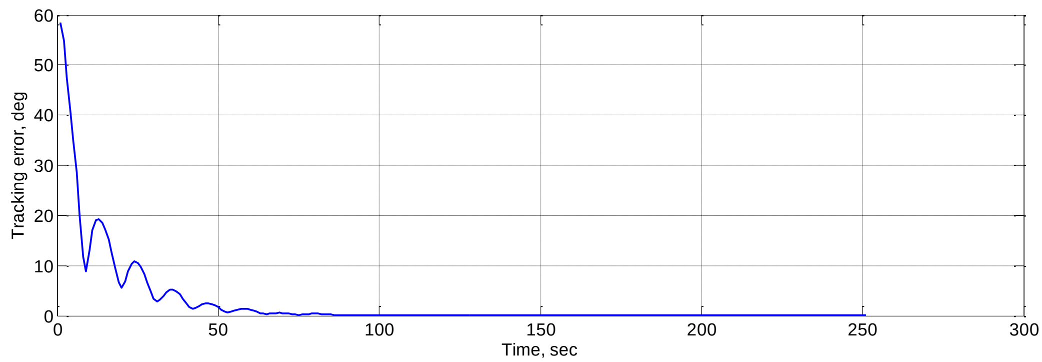

Figure 11. Ground Traget Tracking Error

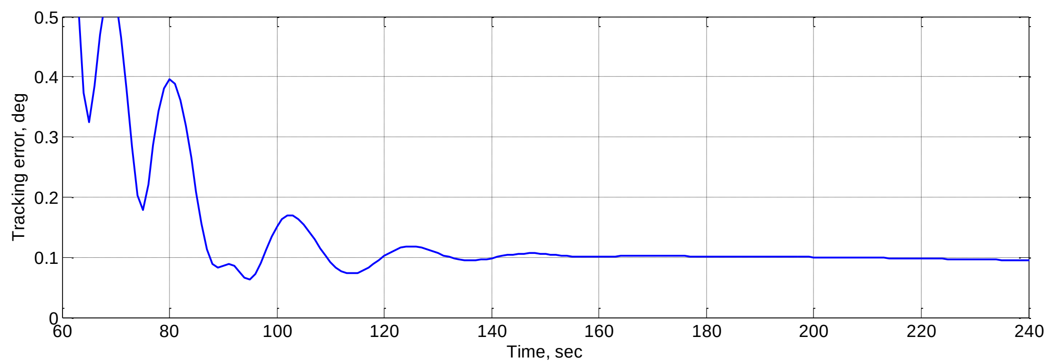

Figure 12. Ground Traget Tracking Error (Zoom in)

The simulation results Figs 11-12 shows that the proposed RW nonlinear controller is not limited to fast tracking with relaxed pointing accuracy. The enhancement in tracking accuracy will be on the expense of the time elapsed to track the fixed ground target and actuator effort.

VIII. Conclusions

In this paper a nonlinear tracking control algorithm is modified and altered to be utilized with exchange momentum actuators (e.g. reaction wheels), for specifically fixed ground target tracking task. The proposed controller cares about fast ground-target tracking. Quick response allows data gathering and downloading in the same communication session for the military intelligence. The proposed RW nonlinear tracking controller is using the commanded attitude rate and acceleration in addition to attitude error and gyroscopic compensation terms. A systematic method for determine a complete kinematics of fixed ground target relative to the orbit frame has been presented. The simulation results show that Rw nonlinear controller is capable to achieve a fast ground target tracking maneuver once the target is being in the satellite FOV. The reaction wheels will apply maximum torque to provide bang-bang control manoeuvres to meet the required target tracking within specified limited time with relaxed pointing accuracy. The wheel momentum is kept well away below the permissible limits. The simulation results also show well robustness behaviour of the proposed controller.The enhancement in tracking accuracy is possible but it will be on the expense of the time elapsed to track the fixed ground target and actuator effort.

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