Blind equalization technique for cross correlation constant modulus algorithm (CC-CMA)

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Blind Equalization Technique for Cross Correlation Constant Modulus Algorithm (CC-CMA)

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Abstract: - Equalization plays an important role for the communication system receiver to correctly recover the symbol send by the transmitter, where the received signals may contain additive noise and intersymbol interference (ISI). Blind equalization is a technique of many equalization techniques at which the transmitted symbols over a communication channel can be recovered without the aid of training sequences, recently blind equalizers have a wide range of research interest since they do not require training sequence and extra bandwidth, but the main weaknesses of these approaches are their high computational complexity and slow adaptation, so different algorithms are presented to avoid this nature.

The conventional Cross Correlation Constant Modulus Algorithm (CC-CMA) suffers from slow convergence rate corresponds to various transmission delays especially in wireless communication systems, which require higher speed and lower bandwidth. To overcome that, several adaptive algorithms with rapid convergence property are proposed based upon the cross-correlation and constant modulus (CC-CM) criterion, namely the recursive least squares (RLS) version of the CC-CMA (RLS-CC-CMA).

This paper proposes a new blind equalization technique, the Exponential Weighted Step-size Recursive Cross Correlation CMA (EXP-RCC-CMA), which is based upon the Exponentially Weighted Step-size Recursive Least Squares (EXP-RLS) and the Recursive Cross Correlation CMA (RCC-CMA) methods, by introducing several assumptions to obtain higher convergence rate, minimum Mean Squared Error (MSE), and hence better receiver performance in digital system. Simulations studies show the rate of convergence, the mean square error (MSE), and the average error versus different signal-to-noise ratios (SNRs) with the other related blind algorithms.

Key-Words: - Blind Equalization, Constant Modulus Algorithm (CMA), Recursive Least Squared (RLS) algorithm, Exponentially Weighted Step-size Recursive Least Squares (EXP-RLS) algorithm, Recursive Cross Correlation Based method for CMA (RCC-CMA) algorithm, Channel Equalization.

1 Introduction

One of the most important advantages of the digital communication system for voice, data and video is their higher reliability in noise handling in comparison with that has the analogue communication property. In modern digital communication systems an estimator of the transmitted symbols represents one of the critical parts of the receiver, it consists typically of an equalizer and a decision device as shown in Fig. 1, and because the equalizer is designed to compensate the channel distortions, through a process known as equalization so it plays an important role for the communication systems. Equalization is a process in which the symbols sent by the transmitter can be recovered correctly from the received signal that suffer from additive noise and the linear channel distortion, known as the Inter-symbol interference (ISI), this means that the transmitted pulses are spreaded out of their limits and overlapped with the adjacent pulses, so on the pulses that correspond to different symbols are not separable and that can severely corrupt the transmitted signal and make it difficult for the receiver to directly recover the send data. Equalization is a process in which the symbols sent by the transmitter can be recovered correctly from the received signal that suffer from additive noise and the linear channel distortion, known as the Inter-symbol interference (ISI). According to the transmission media the main causes for ISI are: the band limited of the cable lines, and the multipath propagation of the cellular communications.

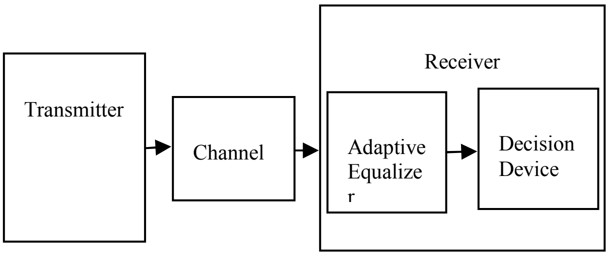

Fig. 1: Digital transmission system using channel equalization.

The conventional or trained adaptive algorithms for equalization use a training signal to update the tap weights of the equalizer's adaptive filter, according to that these methods suffer from consuming large part of the channel bandwidth, i.e., when the pilot or the training sequence is transmitted, no other useful sequences can be transmitted by the same spectrum, and also sometimes it is so difficult to characterize the statistical properties of the training sequence to estimate the radio environment as an example. To overcome these limitations, a lot of researches have been much interest in blind approaches, which can adapt their tap weight vector by restoring certain properties of the transmitted signals' structure that can compensate the transmission of the pilot or the training sequence. So from the above discussion we can classify the equalization techniques into two types which are, blind (non-trained) and non-blind (trained) equalization.

Recent systems (such as GSM system) use well known methods based on training sequences, where a part of signal is known and repeated, and the equalizer is based on matching its output to the reference signal, by adapting its parameters to minimize some criterion (typically MSE). Unfortunately the training sequence consumes a considerable part of the overall message (approx. 25% in GSM) [1]. For this reason, recently much research effort has been devoted to blind equalization algorithms.

Blind equalization or self-recovering algorithms have no training sequence; so they do not require an extra bandwidth, also the bandwidth efficiency potential is increased, and hence the bit rate can be improved [1], but the main weaknesses of these approaches are their high computational complexity and slow adaptation [2][3].

Blind equalization is one of the most important applications of the telecommunication systems, in which the unknown input sequence is recovered from the unknown channel distortion based on the probability roles and statistical properties of the input sequence to the adaptive equalizer, and its performance depends on the characteristics of both the channel and the transmitted sequence [4].

Among all blind equalization algorithms, the constant modulus algorithm (CMA) [5], is a popular, low complexity blind algorithm used for channel equalization and inters symbol interference (ISI) suppression for constant modulus signals [6].

One limitation of the conventional CMA algorithm is that, it is in capable of distinguishing one user data from another in multiuser detection applications; and so on it fails to lock on the desired user signal. By adding a cross-correlation term to the CMA cost function to get the cross correlation CMA (CC-CMA) algorithm presented in to solve such problem in a static multipath channel [7] [8].

However, the classical CC-CMA is found to be ineffective for some applications where a fast convergence is needed (such as radio-mobile system or wireless communication systems) and because the Recursive Least Square (RLS) algorithm is well-known that it has a very fast convergence rate [6], so our proposition is to apply a similar method in order to solve this phenomenon of the CMA.

The rest of the paper is organized as follows. In section 2, the Constant Modulus Algorithm (CMA) is presented. In section 3, the Recursive Cross Correlation CMA (RCC-CMA) method is shown. In section 4, we formulate the principles of the proposed Exponential Weighted Step-size Recursive Cross Correlation CMA (EXP-RCC-CMA) technique and present its parameters that improve the rate of convergence. In section 5, simulation performance results are generated and compared with the conventional CMA, RCC-CMA, and EXP-RCC-CMA algorithms. Finally, we conclude the paper results and simulations in section 6.

2 Channel Equalization

A typical communication system design involves passing the transmitted signal through a communication channel by the transmitter, and at the receiver, the received signal is passed through the receiver components to recover the original signal. However, the channel will affect the transmitted signal because of the channel noise and dispersion which are leading to the Intersymbol Interference (ISI) phenomenon, so it is necessary to pass the received signal at the receiver through an equalizer as shown in Fig. 1, to minimize the channel effect [9][10]. The adaptive equalizer and the decision device at the receiver compensate the Intersymbol Interference (ISI) created by a time dispersive channel as mentioned in the introduction section.

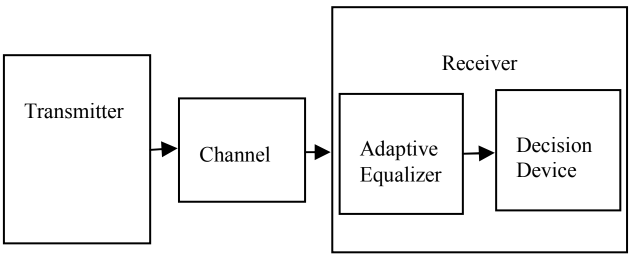

Fig. 2: Digital transmission system using channel equalization.

3 Blind Equalization

Blind equalization is one of the most important applications of the field of telecommunication systems, where the transmitted signal must be isolated from the multipath and interference effects.

Many researches of the blind equalization has been existence for a little over twenty years, during this time the concentration is on developing new algorithms and formulating a theoretical justification for these algorithms. Blind equalization is also known as a self-recovering equalization [4][13][14], and the objective of blind equalization is to recover the unknown input sequence from the unknown channel based on the probability roles and statistical properties of the input sequence to the adaptive equalizer, x(n) as shown in Fig. 2. The receiver can synchronize to the received signal x(n) and to adjust the equalizer filter weights w(n) without the training sequence. The term blind is used in this equalizer because it performs the equalization on the data without a reference signal d(n). Instead, the blind equalizer relies on knowledge of a signal structure and its statistical properties to perform the equalization, the error signal e(n), between the desired and the equalizer output signals, can be used by the adaptive algorithm to update the equalizer filter weights from w(n) to w(n+1) by different approaches until reach the minimum error. The performance of the blind equalization depends on the characteristics of the input signal to the channel and the characteristics of that channel [15].

Many communication signals, such as FM, PM, BPSK, and some QAM signals, have the constant modulus (CM) property, which means the amplitude of the signal is constant after these modulation schemes are applied, the blind equalization for these techniques has been widely used [9][13][14]. However, this CM property is changed after the transmitted signals are corrupted by the channel effects, noise and interference. The CM property can be restored by applying some adaptive algorithms; the source signal can be detected from the channel effects which do not have CM property even without the help of training sequences, these algorithms are generally called Constant Modulus Algorithms (CMA), which is introduced by Godard [1]. So the CMA is used when the transmitted signal possesses the constant envelope property, and the distortion developed to its envelope is used as the measure of a criterion to be minimized.

However, the classical CMA is found to be ineffective for some applications where a fast convergence is needed (such as radio-mobile system) and because the Recursive Least Square (RLS) algorithm is well-known that it has a very fast convergence rate [15], so our proposition is to apply a similar method in order to solve this phenomenon of the CMA.

4 Constant Modulus Algorithm (CMA)

The Constant Modulus Algorithm (CMA) is a special case of Godard algorithm which is a steepest descent algorithm with no training period is present [1]. We can summarize the Godard algorithm equations as the following.

The cost function J_{CM} for the equalization process can be calculated by:

$$J_{CM}(k) = E[(|y(k)|^2 - \alpha_P)^2] \quad (1)$$

where E(.) is the expectation operator, y(k) is the filter output and α_P is called a dispersion constant which is a positive real constant.

The α_P can be obtained from:

$$\alpha_P = \frac{E(|I(k)|^{2p})}{E(|I(k)|^p)} \quad (2)$$

where I(k) is the decision output from the decision device. This cost function's optimization results in the filter coefficients update which equalize only the symbol amplitude, without depending on the carrier phase, and also it is differentiated to the derivative at which an LMS type algorithm is obtained. The resulting update equation for the equalizer coefficients is as below

$$w_i(k+1) = w_i(k) + \mu x(k-i) y_k |y_k|^{p-2} (|y_k|^p - \alpha_p) \quad (3)$$

where μ is a suitable step-size, wi(k) is the i[th] tap of the filter at time k, x(k-i) is the input at time (k-i), p is a positive integer.

The algorithm must stop adaptation when perfect equalization is achieved, so the constant α_P's value results in the gradient of the cost function to be equal to zero, when y(k)=I(k).

The error term for the algorithm is:

$$e(k) = y_k |y_k|^{p-2} (|y_k|^p - \alpha_p) \quad (4)$$

with this error term the CMA algorithm uses the LMS algorithm to update the coefficients, so the resulting update equation for the equalizer coefficients is as below

$$w_i(k+1) = w_i(k) + \mu x(k-i) e(k) \quad (5)$$

where μ is a suitable step-size, wi(k) is the i[th] tap of the filter at time k, x(k-i) is the input at time (k-i).

In the case which p=2, the algorithm is called constant modulus algorithm (CMA).

5 Recursive Cross Correlation Based Method for CMA (RCC-CMA)

The Recursive Cross Correlation Constant Modulus Algorithm (RCC-CMA) relies on the proof work in [16], which the CMA equalizer can be a version of a Minimum Mean Squared Error (MMSE) equalizer. By using the link between CMA and MMSE equalizers, the different transmission delays can be approximated by the CMA equalizer output, and then be found by the RLS algorithm [17].

The MMSE equalizer weights can be approximated by

$$w_i^m = R_{xx}^{-1} R_{xs}^i \approx w_i \quad (6)$$

The autocorrelation matrix of the equalizer input vector [4][17]

$$R_{xx} = E(X(k) X^T(k)) \quad (7)$$

The cross correlation vector between the equalizer input vector and the transmitted signal at delay i [4][17].

$$R_{xs}{}^i = E(X(k) s(k-i)) \quad (8)$$

The estimated autocorrelation and cross correlation matrices can be written as [4].

W_i^m and W_i respectively denote the MMSE and CMA equalizers at delay i. Assuming that the CMA equalizer retrieves the transmission signal with delay i, we form another equalizer such that its output does not have any contribution from s(k-i). Based upon the above relation in (6), we suggest the RLS algorithm for finding the equalizer corresponding to a different delay. The estimated autocorrelation and cross correlation matrices can be written as [4]

$$R_{xx}(k) = \sum_{m=1}^{k} \lambda^{k-i} x(m) x^T(m) \tag{9}$$

$$R_{xs}^i(k) = \sum_{m=1}^{k} \lambda^{k-i} x(m) s(m-i) \tag{10}$$

where λ is a forgetting factor, and isolating the term corresponding to n=k from the rest of the summation on the right hand side of (9) and (10),

$$R_{xx}(k) = \lambda R_{xx}(k-1) + X(k) X^T(k) \tag{11}$$

$$R_{xs}^i(k) = \lambda R_{xs}^i(k-1) + X(k) s(k-i) \tag{12}$$

Let $P(k)=R_{XX}(k)^{-1}$ and using the matrix inversion lemma with (11) and (12), we can write

$$P(k) = \lambda^{-1} P(k-1) - \lambda^{-1} K(k) x^T(k) P(k-1) \tag{13}$$

where

$$K(k) = \frac{\lambda^{-1} P(k-1) x(k)}{1 + \lambda^{-1} x^T(k) P(k-1) x(k)} \tag{14}$$

The update weight vector $w(k)$ equation can be

$$w(k) = w(k-1) + K(k) e(k) \tag{15}$$

6 Exponentially Weighted Step-size RCC-CMA (EXP-RCC-CMA)

In this section, we propose the Exponentially Weighted version of the RCC-CMA, (EXP-RCC-CMA) algorithm, which depends on the estimation method of the inverse exponentially weighted correlation matrix.

The proposed technique in this paper relies on the work done in [18][19][20] by Y. Chen and et. al. for the RCC-CMA algorithm, [17] K.Skowratananont and D.Ratanapanich, [21][22][23][24] by S. Makino and Y. Kaneda for the EXP-RLS algorithm, and by using the link between CMA and MMSE equalizers proved in Zeng's work [29], we can develop the proposed algorithm (EXP-RCC-CMA), where the different transmission delays that can be approximated by the CMA equalizer output, can be treated by the MMSE equalizer that uses the (EXP-RLS) algorithm.

We can summarize the algorithm as the following, as seen in the previous section the autocorrelation matrix of the equalizer input vector

$$R_{xx} = E(X(k) X^T(k)) \tag{16}$$

The cross correlation vector between the equalizer input vector and the transmitted signal at delay i

$$R_{xs}{}^i = E(X(k) s(k-i)) \tag{17}$$

The estimated autocorrelation and cross correlation matrices can be written as [1].

$$R_{xx}(k) = \lambda R_{xx}(k-1) + X(k) X^T(k) \tag{18}$$

$$R_{xs}^i(k) = \lambda R_{xs}^i(k-1) + X(k) s(k-i) \tag{19}$$

where λ is a forgetting factor, and isolating the term corresponding to n=k from the rest of the summation on the right hand side of equation (18) and equation (19), and using the matrix inversion lemma so P(k) can be written as

$$P(k) = \lambda^{-1} P(k-1) - \lambda^{-1} K(k) x^T(k) P(k-1) \qquad (20)$$

To get faster convergence speed and minimum Mean Square Error (MSE), the EXP-RCC-CMA algorithm uses the inverse of the exponentially weighted correlation matrix estimation seen in [19][20][26].

So referring to the work done in [21][22][27][28], each element of the impulse response variation Δw(k) is assumed to be a statistically independent random variable, so the covariance matrix Q(k) of the variation Δw(k) becomes a diagonal matrix, where the diagonal components are the $E[\Delta w_i(k)]$. The $w_0(k)$, which represents the magnitude of the variation, is assumed to take time-invariant value w_0, and based on these assumptions, we set Q(k) as

$$Q(k) = \begin{pmatrix} w_1 & 0 & \ldots\ldots & 0 \\ 0 & w_2 & \ldots\ldots. & 0 \\ & & & \\ 0 & 0 & \ldots\ldots & w_L \end{pmatrix} \qquad (21)$$

where

$$w_i = w_0 \gamma^{i-1} \qquad i = (1, \ldots\ldots, L) \qquad (22)$$

γ: exponential attenuation ratio of the impulse response ($0 < \gamma \le 1$). Elements w_i are time-invariant and decrease exponentially from w_1 to w_L by the same ratio γ as the impulse response w(k). We will define the matrix $P_k(k)$ which is a priori coefficient error covariance matrix,

$$P_k(k) = E[\{w(k) - \hat{w}(k)\}\{w(k) - \hat{w}(k)\}^T] \qquad (23)$$

where $w(k)$ is the filter coefficient vector, $\hat{w}(k)$ is the updated filter coefficient vector, and E[.] is the statistical expectation. Also we will define the matrix $R(k)$ which is a power of the ambient noise n(k),

$$R(k) = E[n(k)^2] \qquad (24)$$

and the vector K(k) is the L^{th} order Kalman gain vector,

$$K(k) = \frac{P_k(k) x(k)}{R(k) + x(k)^T P_k(k) x(k)} \qquad (25)$$

assuming that the ambient noise n(k) is stationary [R(k) ≡ R], we introduce P_{ES} (k) by multiplying the a priori coefficient error covariance matrix P_k (k) of the Kalman filter by (l/R).

$$P_k(k) = R \times P_{ES}(k) \qquad (26)$$

$$P_k(k+1) = P_k(k) + Q(k) \qquad (27)$$

substituting (27) into (26), using R(k) ≡ R, and by using (21), (24), and (26) into (27), we get the following EXP-RLS-CMA algorithm [12][13].

$$\hat{w}(k+1) = \hat{w}(k) + K(k) e(k) \qquad (28)$$

$$K(k) = \frac{P_{ES}(k) x(k)}{1 + x(k)^T P_{ES}(k) x(k)} \qquad (29)$$

$$P_{ES}(k+1) = P_{ES}(k) - k(k) x(k)^T P_{ES}(k) + \frac{Q(k)}{R} \qquad (30)$$

$$e(k) = y(k) - \hat{w}(k)^T x(k) + n(k) \qquad (31)$$

$$Q(k+1) = \lambda Q(k) + x(k) x(k)^T \qquad (32)$$

where

λ is the forgetting factor

$P_{ES}(k)$: L×L matrix

The equation (31) is obtained from the estimate of the exponentially weighted correlation matrix Q(k) by using the matrix inversion lemma. Elements $\{w_1, w_2, ..., w_L\}$ of the Q(k) matrix are not really step-sizes like in the conventional NLMS or the projection algorithms. However these elements function as if they were step-sizes, and from the relationship between the previously proposed ES algorithm [23] and ESP algorithm [24], we call matrix Q(k) a step-size matrix.

On the other hand, the step-size is known to be related to the forgetting factor λ of the RLS algorithm. In fact, according to (30), when the value (Q(k)/R) is large compared to P_{ES}(k), the proportion of P_{ES}(k) in P_{ES}(k+l) becomes small. In other words, old information is forgotten quickly.

The covariance matrix Q(k) of the impulse response variation is added in (27) and (30), according to that the exponentially attenuating bias is always added in diagonal elements of the matrix P_{ES}(k), as a result, the gain vector K(k) attenuates exponentially in (29), the filter coefficient vector w(k) is adjusted by the exponentially attenuating adjustment vector in (28).

7 Results and Simulations

In this section we evaluate the performance of the proposed EXP-RCC-CMA algorithm, where the main parameters of concern are the rate of convergence, the mean square error (MSE), and the average error in dB with different signal to noise ratios, compared with the CMA and RCC-CMA algorithms. In the simulations the transmitted signal s(n) is a QPSK symbol sequence; the different samples are coded with binary sequences taking the two possible values –1 and 1 (s(n) is a sequence of –1s and 1s). The signal to noise ratio of the transmitted sequence SNR=20 dB, the channel is modelled with a FIR filter of third order and the equalizer is realized as a FIR adaptive filter of third order. The CMA, RCC-CMA, and EXP-RCC-CMA algorithms are implemented according to the steps presented in sections 4, 5, and 6. For CMA algorithm μ= 0.08, RCC-CMA algorithm λ = 0.99, and for EXP-RCC-CMA algorithms λ = 0.99, γ = 0.95.

First we will examine the performance of the three algorithms, according to the average error in dB, and the simulation results as the following:

Algorithm	Average Error in Db
CMA	-20.3964
RCC-CMA	-32.1122
EXP-RCC-CMA	-39.1067

Table 1. EXP-RCC-CMA versus CMA and RCC-CMA.

From Table 1, Fig.3 and Fig.4 we can see that EXP-RCC-CMA has the fastest convergence rate compared with the other algorithms, and also has the best performance by achieving the highest average error in dB for different values of signal to noise ratio (SNR), and its MSE is much lower than the CMA algorithm.

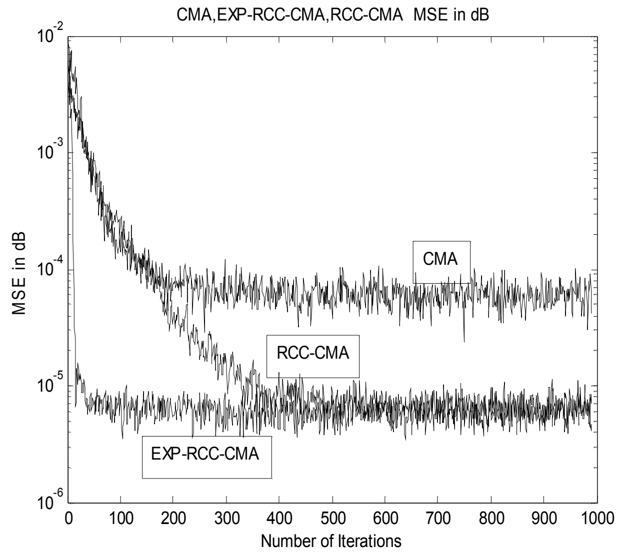

Fig.2. EXP-RCC-CMA versus RCC-CMA and CMA Mean Square Error (MSE).

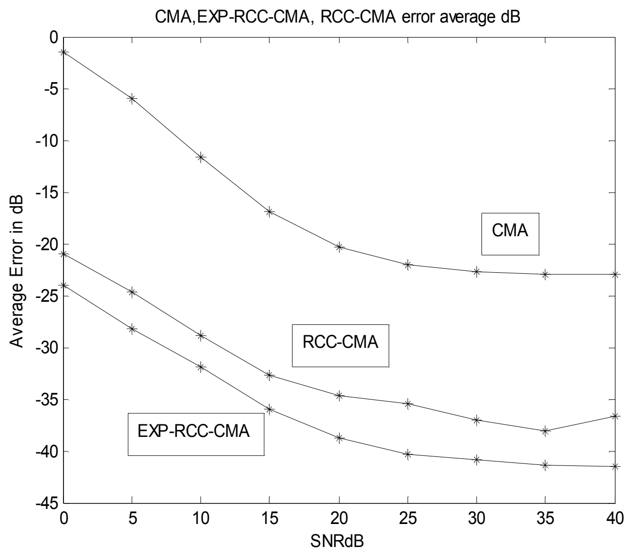

Fig.3. EXP-RCC-CMA versus RCC-CMA and CMA Average Error with SNR.

Fig. 5 shows the comparison of the three algorithms with respect to the absolute error, and from it we can see that the EXP-RCC-CMA algorithm provides better performance than the other algorithms. The maximum value of the absolute error of the EXP-RCC-CMA algorithm not exceed 0.5 V, while the values of the RCC-CMA and the CMA algorithms exceed 2 V.

The absolute error of the EXP-RCC-CMA algorithm is settled after a few samples not exceed 10 samples, but for the RCC-CMA algorithm after 60 samples and the CMA algorithm after 80 samples. Those results lead fastest convergence rate and minimum absolute error of the EXP-RLS-CMA algorithm rather than the other algorithms.

The main advantages of the proposed algorithm could be summarized as follows, sufficient robustness in fixed-point applications, minimum mean square error (MSE) as shown from Fig.3, minimum average error with lower signal to noise ratios (SNR) as shown from Fig.4, and minimum absolute error and high adaptation rate as shown from Fig.5. Our previous work showed that the variation of a channel impulse response becomes progressively smaller by the same exponential ratio as the impulse response.

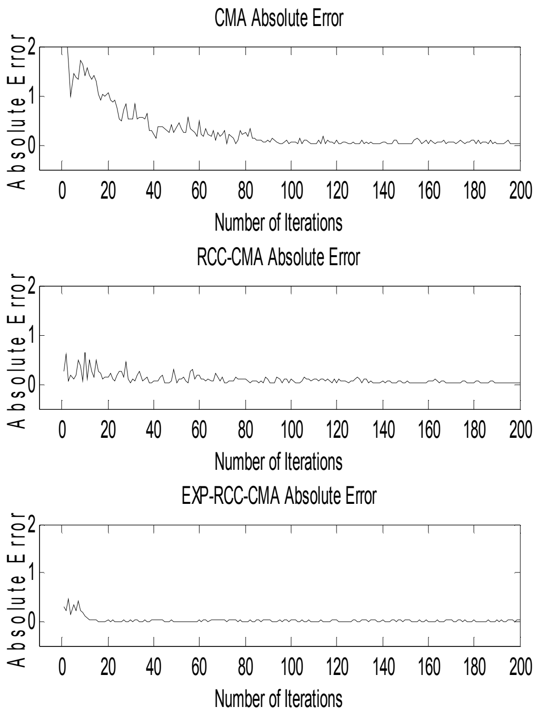

Fig.5. Absolute error simulation results.

8 Conclusion

This paper introduces a new blind equalization technique, the EXP-RLS-CMA algorithm, which is simulated and tested for noise minimization in QPSK signal or 4 QAM symbol sequences, the obtained results show that the EXP-RLS-CMA algorithm is very promising, and has an improved performance by providing a double convergence speed compared to the conventional CMA, and the RLS-CMA algorithms.

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