Gaining-sharing knowledge based algorithm for solving optimization problems: a novel nature-inspired algorithm

Article *in* International Journal of Machine Learning and Cybernetics · December 2019

DOI: 10.1007/s13042-019-01053-x

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International Journal of Machine Learning and Cybernetics
https://doi.org/10.1007/s13042-019-01053-x

ORIGINAL ARTICLE

Gaining-sharing knowledge based algorithm for solving optimization problems: a novel nature-inspired algorithm

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Received: 12 April 2019 / Accepted: 11 December 2019

Abstract

This paper proposes a novel nature-inspired algorithm called Gaining Sharing Knowledge based Algorithm (GSK) for solving optimization problems over continuous space. The GSK algorithm mimics the process of gaining and sharing knowledge during the human life span. It is based on two vital stages, junior gaining and sharing phase and senior gaining and sharing phase. The present work mathematically models these two phases to achieve the process of optimization. In order to verify and analyze the performance of GSK, numerical experiments on a set of 30 test problems from the CEC2017 benchmark for 10, 30, 50 and 100 dimensions. Besides, the GSK algorithm has been applied to solve the set of real world optimization problems proposed for the IEEE-CEC2011 evolutionary algorithm competition. A comparison with 10 state-of-the-art and recent metaheuristic algorithms are executed. Experimental results indicate that in terms of robustness, convergence and quality of the solution obtained, GSK is significantly better than, or at least comparable to state-of-the-art approaches with outstanding performance in solving optimization problems especially with high dimensions.

Keywords Evolutionary computation · Global optimization · Meta-heuristics · Nature-inspired algorithms · Population-based algorithm

1 Introduction

Optimization is the process of finding the best combination for a set of decision variables to solve a certain problem. Optimization arises in many fields, different disciplines and countless applications [1]. Hard optimization problems arise in huge number of applications in real life. In which, it is very hard to reach the global optimum solution. Therefore, many algorithms tried to tackle this kind of problems. The algorithms could be classified as *exact* methods and *approximate* methods. Exact methods are guaranteed to obtain an optimal solution in a reasonable time unless the problem is classified as NP-Hard problem, in which there is no polynomial time exists. This leads to very high computational time. Thus, the use of approximate methods has gained much more attention during the last three decades. In approximate methods, the main target is to obtain a satisfactory solution in a reasonable time.

A new kind of approximate algorithms has been developed [2] known as metaheuristic. Metaheuristic is widely used over the last three decades because of its simplicity, ease of implementation, ability to avoid local optima and it deals with derivative free problems. Two important characteristics of metaheuristic are *exploration* and *exploitation*. The former one indicates the ability of the algorithm to discover new search areas, while the later one focus on finding the best solution in a promising region of the search space. The successful metaheuristic is the one that is able to provide the balance between *exploration* and *exploitation*.

Electronic supplementary material The online version of this article (https://doi.org/10.1007/s13042-019-01053-x) contains supplementary material, which is available to authorized users.

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In the literature, it can be found different classifications for metaheuristic [3]. *Nature inspired* vs. *non-nature inspired*, *population based* vs. *single point search*, *dynamic objective function* vs. *static objective function*, *one single neighborhood* vs. *various neighborhood* and *memory usage* vs. *memory less* methods.

The literature can be divided into three main directions: improving the current methods by controlling the parameters of the algorithms, hybridizing different algorithms to benefit from each one, and introducing a new algorithm.

Introducing a novel algorithm for solving optimization problems have attracted many researchers during the last three decades. Therefore, a *new* classification for the source of inspiration is introduced through the rest of this section and a detailed review is presented. The source of inspiration for nature inspired algorithms can be classified into four groups. As depicted in Fig. 1.

- *Evolutionary techniques* that is inspired from biology. In evolutionary algorithm there is an initial random population that evolve over generations to produce new solutions by means of crossover and mutation and eliminate the worst solutions in order to improve the fitness value.

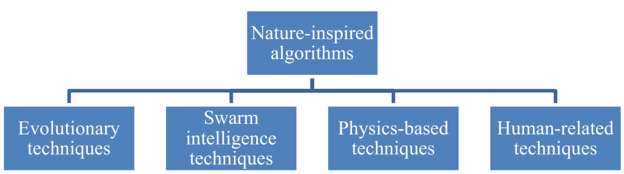

Fig. 1 Classification of nature inspired algorithms

- *Swarm Intelligence techniques* that is inspired from the behavior of social insects or animals, nature inspired algorithms. In swarm intelligence, every individual has its own intelligence and behavior, but the integration of the individuals gives more power to solve complex problems.
- *Physics based techniques* that is inspired by the rules governing a natural phenomenon.
- *Human related techniques* that is inspired from the human being. Every individual does physical activities (body activities) that affect his performance *and* non-physical activities like thinking and behavior (mind activities).

Algorithms belonging to each category are listed in Tables 1, 2, 3 and 4 respectively. Each table has the algorithm, author, year and the number of citations till 20 Dec 2018.

From the above tables, Figs. 2, 3, 4 and 5 are presented to demonstrate how meta-heuristic algorithms evolved during the last three decades. Figures 2 and 3 present the number of algorithms in each category and the share of each category in the meta-heuristic algorithms. It is obvious that the Swarm Intelligence algorithms is the leading category. Figure 4 demonstrates the quality of each category by using the total number of citations in each category. It can be seen that the evolutionary algorithms are the most commonly used in the meta-heuristic algorithms and leading the total number of citations. Figure 5 presents how the mat-heuristic algorithms evolved over time in the last three decades. It is clear that the evolutionary algorithms were the leader for the meta-heuristic algorithms since 1966 and till today there are many algorithms

Table 1 List of evolutionary algorithms

Algorithm	Author	Year	Citations	References
Evolutionary programming	Fogel et al.	1966	4418	[4]
Evolution strategy	Rechenberg	1973	4397	[5]
Genetic algorithms	Holland	1975	60,347	[6]
Tabu search	Glover	1986	4627	[2]
Co-evolving algorithm	Hillis	1990	1382	[7]
Cultural algorithm	Reynolds	1994	970	[8]
Genetic Programming	Koza	1994	18,989	[9]
Estimation of distribution algorithm	Mühlenbein and PaaB	1996	1172	[10]
Differential evolution	Storn and Price	1997	19,448	[11]
Grammatical evolution	Ryan et al.	1998	642	[12]
Gene expression	Ferreira	2001	2108	[13]
Quantum evolutionary algorithm	Han and Kim	2002	1459	[14]
Imperialist competitive algorithm	Gargari and Lucas	2007	1637	[15]
Differential search algorithm	Civicioglu	2011	258	[16]
Backtracking optimization algorithm	Civicioglu	2013	437	[17]
Stochastic fractal search	Salimi	2014	130	[18]
Synergistic fibroblast optimization	Dhivyaprabha et al.	2018	0	[19]

Table 2 List of swarm intelligence algorithms

Algorithm	Author	Year	Citations	References
Memetic algorithm	Moscato	1989	2019	[20]
Ant colony optimization	Dorigo	1992	13,086	[21]
Continuous particle swarm optimization	Eberhart and Kennedy	1995	13,504	[22]
Particle swarm algorithm	Kennedy and Eberhart	1995	5484	[23]
Binary particle swarm optimization	Kennedy and Eberhart	1997	4839	[24]
Artificial immune system	Castro and Timmis	2002	2765	[25]
Clonal selection algorithm	Castro and Zuben	1999	1291	[26]
Self-organizing migrating algorithm	Zelinka	2000	317	[27]
Marriage in honey bees	Abbass	2001	406	[28]
Artificial fish swarm algorithm	LI et al.	2002	67	[29]
Bacterial foraging	Passino	2002	2738	[30]
Bee dance algorithm	Gordon et al.	2003	44	[31]
Bees swarm optimization heuristic algorithm	Lučić and Teodorović	2003	198	[32]
Queen-bee evolution	Jung	2003	131	[33]
Shuffled frog leaping algorithm	Eusuff and Lansey	2003	1348	[34]
Beehive algorithm	Wedde et al.	2004	258	[35]
Bee colony optimization	Teodrovic, Dell' Orco	2005	421	[36]
Cooperative bees swarm optimization	Drias et al.	2005	157	[37]
Glowworm swarm optimization	Krishnanand and Ghose	2005	342	[38]
Honey bee swarm optimization algorithm	Karaboga	2005	5009	[39]
Virtual bee algorithm	Yang	2005	349	[40]
Bees algorithms	Pham et al.	2006	1296	[41]
Cat swarm optimization	Chu et al.	2006	317	[42]
Invasive weed optimization	Mehrabian and Lucas	2006	876	[43]
Termite swarm algorithm	Roth and Wicker	2006	37	[44]
Virtual ant algorithm	Yang et al.	2006	13	[45]
Artificial bee colony	Karaboga and Basturk	2007	4148	[46]
Bacterial-GA foraging	Chen et al.	2007	31	[47]
Good lattice swarm optimization	Su et al.	2007	159	[48]
Monkey algorithm	Zhao, Tang	2008	89	[49]
Accelerated PSO	Yang	2008	161	[50]
Biogeography-based optimization	Simon	2008	2093	[51]
Fast bacterial swarming algorithm	Chu et al.	2008	71	[52]
Fish-school search	Filho et al.	2008	121	[53]
Roach infestation algorithm	Havens et al.	2008	82	[54]
Bumblebees algorithm	Comellas and Navarro	2009	16	[55]
Cuckoo search	Yang and Deb	2009	3439	[56]
Group search optimizer	He et al.	2009	562	[57]
Paddy field algorithm	Premaratne et al.	2009	46	[58]
Bat algorithm	Yang	2010	2322	[59]
Consultant-guided search	Iordache	2010	30	[60]
Eagle strategy	Yang and Deb	2010	157	[61]
Firefly algorithm	Yang	2010	1225	[62]
Hierarchical swarm model	Chen et al.	2010	26	[63]
Termite colony optimization	Hedayatzadeh et al.	2010	38	[64]
Eco-inspired evolutionary algorithm	Parpinelli and Lopes	2011	23	[65]
Fruit fly optimization algorithm	Pan	2011	766	[66]
Weightless swarm algorithm	Ting et al.	2011	12	[67]
Artificial cooperative search algorithm	Civicioglu	2012	73	[68]
Flower pollination algorithm	Yang	2012	740	[69]
Japanese tree frogs calling algorithm	Hern´andez and Blum	2012	31	[70]

Table 2 (continued)

Algorithm	Author	Year	Citations	References
Krill herd algorithm	Gandomi and Alavi	2012	787	[71]
The Great Salmon run algorithm	Mozaffari	2012	35	[72]
The OptBees algorithm	Maia et al.	2012	27	[73]
Wolf search algorithm	Tang et al.	2012	116	[74]
Dolphin echolocation	Kaveh and Farhoudi	2013	164	[75]
Egyptian vulture optimization algorithm	Sur et al.	2013	24	[76]
Swallow swarm optimization algorithm	Neshat et al.	2013	31	[77]
Animal migration optimization	Li et al.	2014	123	[78]
Chicken swarm optimization	Meng et al.	2014	142	[79]
Grey wolf optimizer	Mirjalili et al.	2014	1591	[80]
Ant lion optimizer	Mirjalili	2015	476	[81]
Artificial algae algorithm	Uymaza et al.	2015	50	[82]
Bird swarm algorithm	Meng et al.	2015	67	[83]
Dragonfly algorithm	Mirjalili	2015	291	[84]
Virus colony search	Li et al.	2015	48	[85]
Crow search algorithm	Askarzadeh	2016	202	[86]
Dolphin swarm optimization algorithm	Yong et al.	2016	2	[87]
Shark smell optimization	Oveis et al.	2016	52	[88]
Whale optimization algorithm	Mirjalili and Lewis	2016	538	[89]
Butterfly-inspired algorithm	Qi et al.	2017	4	[90]
Grasshopper optimization algorithm	Saremi et al.	2017	124	[91]
Mouth brooding fish algorithm	Jahani and Chizari	2017	2	[92]
Salp swarm algorithm	Mirjalili et al.	2017	102	[93]
Selfish herd optimizer	Fausto et al.	2017	8	[94]
Spotted hyena optimizer	Dhiman and Kumar	2017	22	[95]
Squirrel search algorithm	Jain et al.	2018	3	[96]

that are presented in the evolutionary category. During the last two decades, the Swarm Intelligence gained more attention from the researchers and competes strongly with the Evolutionary algorithms and Physics based algorithms. The human based algorithms start to gain attention as a new trend in the last decade, but still cannot compete with the evolutionary algorithms, Swarm Intelligence and physics-based algorithms.

Regarding the above discussion, it is obvious that there are very few efforts in developing a new human-base algorithm. Therefore, a novel nature inspired algorithm based on human is presented.

The rest of this paper is organized as follows, Sect. 2 presents in details the novel proposed algorithms. Numerical experiments and comparisons are presented in Sect. 3. Finally, the paper is concluded in Sect. 4.

2 Proposed algorithm: gaining-sharing knowledge based algorithm (GSK)

Gaining-Sharing knowledge optimization algorithm (GSK) is based on the philosophy of gaining and sharing knowledge during the human life span. It is based on two vital stages, the first stage is called beginning-intermediate or junior gaining and sharing phase and the second stage is called intermediate-expert or senior gaining and sharing phase. These two phases are described in the following, respectively.

Virtually, all individuals (persons) in a specific population can interact together and they continuously influence each other through cooperation and competition to be very experienced and qualified enough to deal with real-life situations and solve complex problems. However, to be experienced person, you have to gain and share knowledge from/with others. Therefore, during the human life span, each person in a specific population gains knowledge at early stage (early-middle years) in which gaining knowledge through different types of very small networks such as his/her (family, neighbors, relatives) is more realistic than gaining it from different types of larger networks such as (work, social, friends and many others). Besides, due to limited experience as there is very few sources of knowledge during this stage, he/she still has a desire to share his opinion, thoughts and skills with other different types of people that maybe outside small networks. Actually, during this stage, it must be taken into consideration that he is not qualified

Table 3 List of physics-based algorithms

Algorithm	Author	Year	Citations	References
Micro-canonical annealing algorithm	Creutz	1983	605	[97]
Simulated annealing	Kirkpatrick et al.	1983	43,924	[98]
Stochastic diffusion search	Bishop	1989	172	[99]
Self-propelled particles	Vicsek et al.	1995	5083	[100]
Variable neighborhood algorithm	Mladenovic and Hansen	1995	3423	[101]
Predatory search	Linhares	1998	38	[102]
Photosynthetic algorithm	Murase	2000	22	[103]
Harmony search	Geem et al.	2001	4231	[104]
Gravitational search optimization algorithm	Webster and Bernhard	2003	29	[105]
Big bang–big crunch-based optimization	Erol and Eksin	2005	710	[106]
Central force optimization	Formato	2007	229	[107]
Intelligent water drop	Shah-Hosseini	2007	263	[108]
River formation dynamics	Rabanal et al.	2007	103	[109]
Slime mold algorithm	Monismith and Mayfield	2008	23	[110]
Gravitational search algorithm	Rashedi et al.	2009	2927	[111]
Charged system search	Kaveh and Talatahari	2010	641	[112]
Electro-magnetism optimization	Cuevas et al.	2011	82	[113]
Galaxy-based search algorithm	Shah-Hosseini	2011	89	[114]
Spiral optimization	Tamura and Yasuda	2011	37	[115]
Black hole algorithm	Hatamlou	2012	371	[116]
Curved space optimization	Moghaddam et al.	2012	21	[117]
Ray optimization	Kaveh and Khayatazad	2012	200	[118]
Water cycle algorithm	Eskandar et al.	2012	331	[119]
Atmosphere clouds model	Yan et al.	2013	12	[120]
Mine blast algorithm	Sadollaha et al.	2013	226	[121]
Colliding bodies optimization	Kaveh and Mahdavi	2014	234	[122]
Kinetic energy	Moein and Logeswaran	2014	20	[123]
Lightning search algorithm	Shareef et al.	2015	71	[124]
Weighted superposition attraction	Adil and Akpinar	2015	11	[125]
A sine cosine algorithm	Mirjalilia	2016	239	[126]
Multi-verse optimizer	Mirjalili et al.	2016	221	[127]
Electro-search algorithm	Tabari and Ahmad	2017	11	[128]
Lightning attachment procedure optimization	Nematollahi et al.	2017	5	[129]
Thermal exchange optimization	Kaveh and Dadras	2017	26	[130]
Find-fix-finish-exploit-analyze (F3EA)	Kashan et al.	2018	0	[131]

Table 4 List of human related algorithms

Algorithm	Author	Year	Citations	References
Society and civilization	Ray and Liew	2003	336	[132]
Human-inspired algorithm	Zhang et al.	2009	20	[133]
League championship algorithm	Kashan	2009	88	[134]
Social emotional optimization	Xu et al.	2010	23	[135]
Brain storm optimization	Shi	2011	221	[136]
Teaching–learning-based optimization	Rao et al.	2011	791	[137]
Anarchic society optimization	Shayeghi and Dadashpour	2012	26	[138]
Volleyball premier league algorithm	Moghdani and Salimifard	2018	1	[139]

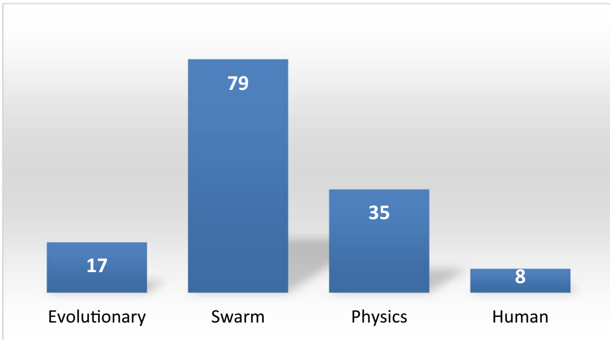

Fig. 2 Number of algorithms in each category

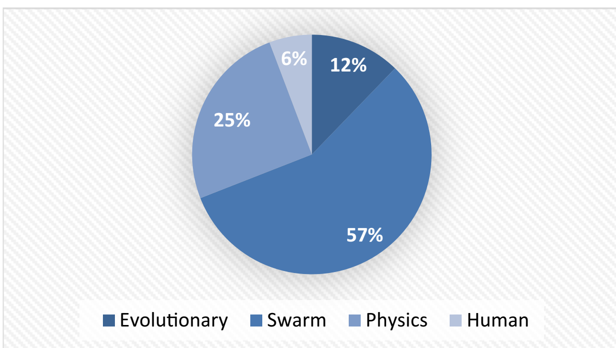

Fig. 3 Percentage share for each category in the overall meta-heuristic algorithms

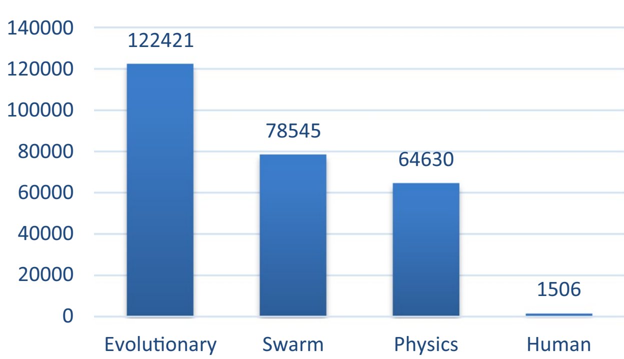

Fig. 4 Total number of citations for each category

enough with very little experience in life to differentiate and classify different types of people, that does not belong to his/her small networks, into good and bad persons.

On the other hand, during (middle-later) years stage, due to interacting with larger networks such as (work colleagues, social media friends and others), each person has its own knowledge in different fields that can be significantly enhanced by gaining it from others which is mainly derived by following leaders' success and believe on opinions of elite persons in addition to avoiding failure persons or people with radical concept or bad performance in all fields. In fact, during this stage, each one has a great ability to judge, think and classify different types of people into good, medium and bad classes. Thus, he can easily share his knowledge and experience in different fields with the most appropriate persons with good characteristics and behaviors (i.e. to benefit from their knowledge and experience). The mathematical explanation of the aforementioned concept of gaining-sharing knowledge is presented below.

Let $x_i, i = 1, 2, 3, \ldots, N$ be the persons of a specific population, i.e., this population contains N persons and each person x_i is defined by $x_{ij} = (x_{i1}, x_{i2}, \ldots, x_{iD})$, where D is the number of fields of disciplines i.e. branch of knowledge assigned to a person which determines the dimensions of a person and f_i, $i = 1,2,\ldots,$N are their corresponding fitness values, respectively.

Thus, it can be obviously deduced from Fig. 6 that the main idea is that during junior gaining and sharing phase (early-middle stage) the number of dimensions of each vector will be replaced (changed) by another values using junior gaining-sharing scheme is more than the number of updated dimensions using senior gaining and sharing scheme i.e. the number of updated dimensions using junior gaining and sharing rule is more than the number of updated dimensions using senior gaining-sharing scheme. However, during senior phase (middle-later stage), the number of updated dimensions of each vector using senior gaining and sharing scheme is more than the number of updated dimensions using junior gaining and sharing scheme. Besides, it must be taken into consideration that the required number of dimensions that will be replaced using both junior and senior phases depends on the value of knowledge rate that controls the volume of knowledge that will be transferred during generations from others using junior and senior gaining-sharing knowledge schemes.

Therefore, the number of the desired number of dimensions that will be updated or changed (using junior scheme) and the other number of dimensions that will be updated (using senior scheme) during generations must be determined for each vector at the beginning of the search. Based on the fundamental concept of gaining-sharing knowledge, the number of dimensions D is determined using the following non-linear decreasing and increasing formula (experience equation). Note that during the generations of optimization process, the number of dimensions that will updated using junior scheme is decreased while the number of dimensions that will be updated by senior scheme is decreased:

$$D(juniorphase) = (problemsize) \times \left(1 - \frac{G}{GEN}\right)^k, \tag{1}$$

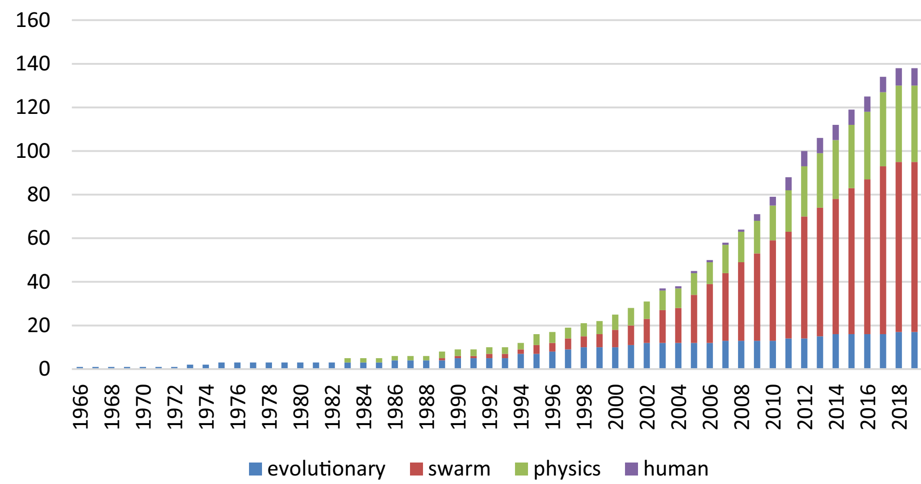

Fig. 5 Evolution of each category over the last three decades

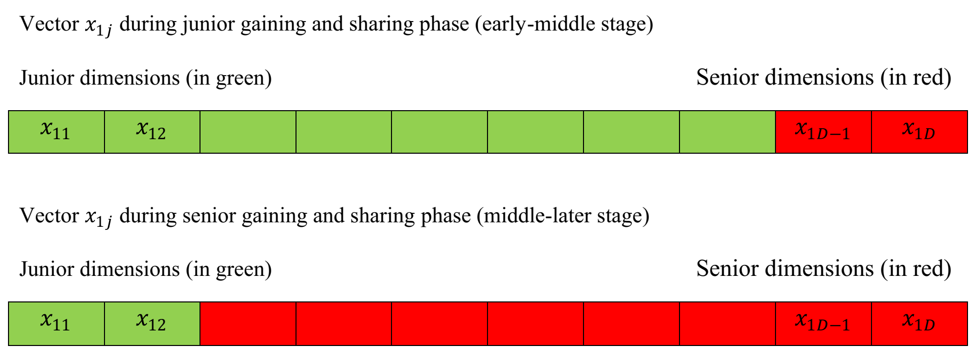

Fig. 6 Vector representation during junior and senior gaining-sharing knowledge phases

Table 5 Number of dimensions updated using junior and senior schemes with problem size 100 and K = 2

G=	G/GEN=	D: number of dimensions updated using both (junior and senior schemes)
0	0	D(junior) = 100, D(junior) = 0
0.25 GEN	0.25	D(junior) = 57, D(senior) = 43
0.5 GEN	0.5	D(junior) = 25, D(senior) = 75
0.75	0.75	D(junior) = 7, D(senior) = 93
GEN	1	D(junior) = 0, D(senior) = 100

where k is KNOWLEDGE rate which is a real number > 0, G is generation number and GEN is the maximum number of generations:

$$D(seniorphase) = problemsize - D(juniorphase).$$

Thus, the number of gained and shared dimensions for each vector using both schemes will be determined at initialization phase. For more clarification, assume that the problem size is 100 and k is 2. Thus, this equation is decreased in increasing rate (Table 5).

Obviously, the knowledge rate k controls the experience rate for each individual through generations using both schemes (from junior phase to senior phase). If $k = 1$, it is linearly decreased and increased i.e. the number of updated Dimensions using junior scheme is linearly decreased while the number of updated Dimensions using senior scheme is linearly increased through generations, otherwise it is non-linearly decreased and increased, respectively. For $k \in (0,1)$, the number of updated dimensions using junior scheme is decreased (non-linearly) slowly. Thus, during generation, the junior scheme will be applied more than senior scheme which means experience acquired in different branches of knowledge with slow rate. On the other hand, For $k > 1$, the number of updated dimensions using junior scheme is decreased (non-linearly) rapidly. Therefore, during generations, the senior scheme will be used more than junior scheme which means experience acquired in different branches of knowledge with fast rate.

2.1 Junior gaining-sharing knowledge phase

In this phase, each individual tries to gain knowledge from the closest and trusted individuals that belong to small groups while he also tries to share knowledge with some individual who does not belong to or, is not member in any group due to his curiosity and desire of exploring others.

Thus, the updating of each individual can be computed using junior scheme as follows:

1. Arrange all individuals in ascending order according to their objective function value: $x_{best}, \dots \dots, x_{i-1}, x_i, x_{i+1}, \dots \dots x_{worst}$
2. Then, for each individual x_i, select two different individuals (the closest individuals), the nearest better (x_{i-1})and worsen individuals (x_{i+1}) than current one to constitute the gain source of knowledge. Besides, select another individual randomly (x_r) to be the source of sharing knowledge. The pseudo code of junior gaining-sharing knowledge phase is presented in Fig. 7.

Note that: in this phase, the best and worst individuals are updated by using the closest best two individuals and the closest worsen two individuals, respectively.

If x_i is the global best, select the nearest forward best two individuals as follows: (x_{best}, x_{best+1}, x_{best+2}).

If x_i is the global worst, select the nearest former worsen two individuals as follows: ($\dots$ $\dots$ $\dots$ $x_{worst-2}$, $x_{worst-1}$,x_{worst}).

Where k_f is which is a real number > 0. It is the (knowledge factor) that controls the total amount of gained and shared knowledge that will be added from others to the current individuals during generations.

Where $k_r \in [0, 1]$. It is the (knowledge ratio) that controls the total amount of gained and shared knowledge that will be transferred (inherited) during generations (the ratio between the current and acquired experience).

2.2 Senior gaining-sharing knowledge phase

This phase is concerned with utilization of available information and appropriate knowledge from different categories of the persons within specific population i.e. best, better and worst persons. The utilization means the impact and effect of others (good and bad persons) on a person. Thus, the updating of each individual can be computed using senior scheme as follows:

1. After sorting all individuals on ascending order according to their objective function, they will be divided into three category best individuals, better or middle individuals, worst individuals.

Best people	Better people	Worst people
$100p\%$ (x_{p-best})	$N-(2\times 100p\%)$ (x_m)	$100p\%$ ($x_{p-worst}$)

2. Then, for each individual x_i, the proposed senior scheme uses two random chosen vectors of the top and bottom $100p\%$ individuals in the current population of size NP to form the gaining part while the third vector is selected randomly from the middle $N-(2\times 100p\%)$ individuals to form the sharing part. The pseudo code of senior gaining-sharing knowledge phase is presented in Fig. 8.

where p ∈ [0,1], and p = 0.1, 10% of NP is suitable.

The pseudo code and the flow chart of GSK algorithm is presented in Figs. 9 and 10, respectively.

3 Numerical experiments and comparisons

In this section, the computational results of GSK are discussed along with comparisons with other state-of-the-art algorithms.

3.1 Experiments setup

The performance of the proposed GSK algorithm was tested on 30 benchmark functions proposed in the CEC 2017 special session on real-parameter optimization. A detailed description of these test functions can be found in [140]. These 30 test functions can be divided into four classes:

For i=1:NP
For j=1:D
    If rand<=k_r (Knowledge ratio)
      If $f(x_i) > f(x_r)$
        x_{ij}^{new}=$x_i + k_f$*[(x_{i-1}-x_{i+1}) + (x_r - x_i)]
      else
        x_{ij}^{new}=$x_i + k_f$*[(x_{i-1}-x_{i+1}) + (x_i - x_r)]
      End(if)
    Else $x_{ij}^{new} = x_{ij}^{old}$
    End (If)
  End (for j)
End (for i)

Fig. 7 Pseudo code of junior gaining-sharing knowledge phase

For i=1:NP
For j=1:D
    If rand<=k_r (Knowledge ratio)
      If $f(x_i) > f(x_m)$
        x_{ij}^{new}=$x_i + k_f$*[($x_{p-best} - x_{p-worst}$) + ($x_m - x_i$)]
      else
        x_{ij}^{new}=$x_i + k_f$*[($x_{p-best} - x_{p-worst}$) + ($x_i - x_m$)]
      End(if)
    else $x_{ij}^{new} = x_{ij}^{old}$
    End (If)
  End (for j)
End (for i)

Fig. 8 Pseudo code of senior gaining-sharing knowledge phase

Fig. 9 Pseudo code of GSK algorithm

Begin
G=0, initialize parameters: N, k_f , k_r, K and P.
Create a random initial population $x_i, i = 1,2, \ldots\ldots N$
Evaluate $f(x_i)$, $\forall i, i = 1,2, \ldots\ldots N$
For G=1 to GEN
Compute the number of (Gained and Shared dimensions of both phases) using experience eq. (1);
//Junior gaining-sharing knowledge phase //
//Senior gaining-sharing knowledge phase//
If $f(x_i^{new}) \le f(x_i^{old})$, $x_i^{old} = x_i^{new}$, $f(x_i^{old}) = f(x_i^{new})$ end // update each vector
If $f(x_i^{new}) \le f(x_{best}^G)$, $x_{best}^G = x_i^{new}$, $f(x_{best}^G) = f(x_i^{new})$ end // update global best
End For…. N
End For…. G
End For…Begin

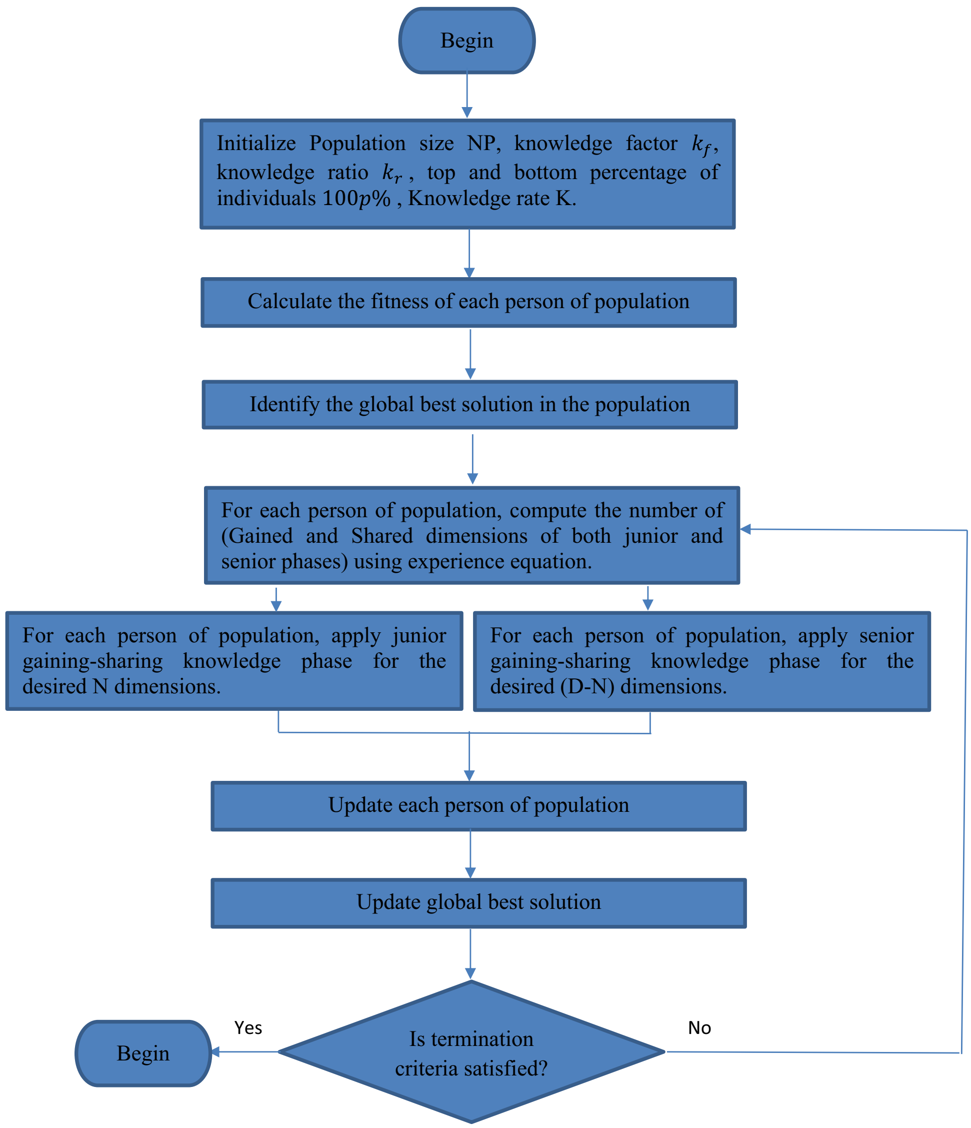

Fig. 10 The flow chart of GSK algorithm

1. Unimodal functions f1–f3;
2. Simple multimodal functions f4–f10;
3. Hybrid functions f11–f20;
4. Composition functions f21–f30;

Besides, CEC2011 consists of 22 real-world optimization problems for the CEC2011 special session and competition on real world optimization problems. A detailed description of these test functions can be found in [141].

3.2 Parameter settings and involved algorithms

To evaluate the performance of algorithms, experiments were conducted on the test suite. We adopt the solution error measure $(f(x) - f(x^*))$, where x is the best solution obtained by algorithms in one run and x^* is well-known global optimum of each benchmark function. Error values and standard deviations smaller than 10–8 are taken as zero [140]. For CEC2017, the dimensions (D) of function are 10, 30, 50 and 100, respectively. The maximum number of function evaluations (FEs), the terminal criteria, is set to $10{,}000 \times D$, all experiments for each function and each algorithm run 51 times independently. For CEC2011, the problems have different dimensions [141]. The maximum number of function evaluations (FEs), the terminal criteria, is set to 150,000, all experiments for each function and each algorithm run 25 times independently. GSK are compared with 10 state-of-the-art population-based algorithms in relevant literature. These algorithms are: TLBO [137], GWO [80], SFS [18], AMO [78], DE [11], BBO [51], ACO [21], ES [30], GA [6], PSO [24]. The control parameters of the mentioned algorithms are given below in Table 6. Note that the control parameters of all algorithms were directly taken from their original references. To the best of our knowledge, this is the first study that uses all these different types of approaches to carry out evaluation and comparisons on up-to-date benchmark problems.

To perform comprehensive evaluation, the presentation of the experimental results is divided into two subsections. First, the performance of the proposed algorithm is discussed. Second, an overall performance comparison between GSK and other 10 state-of-the-art algorithms is provided.

The Performance assessment of the different algorithms is based on score metric which is recently defined for the CEC 2017 competition [140]. Thus, the evaluation method for each algorithm is based on a score of 100 which is based on two criteria as follows taking into account higher weights will be given for higher dimensions:

1. 50% summation of error values for all dimensions as follows:

$$SE = 0.1 \times \sum_{i=1}^{29} ef_{10D} + 0.2 \times \sum_{i=1}^{29} ef_{30D} + 0.3 \times \sum_{i=1}^{29} ef_{50D} + 0.4 \times \sum_{i=1}^{29} ef_{100D}.$$

$$\text{Score } 1 = \left(1 - \frac{SE - SE_{min}}{SE}\right) \times 50.$$

Table 6 The control parameters of search algorithms

Name	Specifications	NP
TLBO	No special parameters	50, the population size is 50 because of this algorithm has two phases
GWO	a = 2-2(g/max_g)	100
SFS	Maximum diffusion number (MDN) is set to 1	50, the population size is 50 because of this algorithm has two phases
AMO	The number of animals in each group was set to 5	50, the population size is 50 because of this algorithm has two phases
DE	F = 0.5, CR = 0.9 in accordance	100
BBO	Habitat modification probability = 1, immigration probability bounds per gene = [0,1], step size for numerical integration of probabilities = 1, maximum immigration and migration rates for each island = 1, and mutation probability = 0	100
ACO	Initial pheromone value = 1e−6; pheromone update constant = 20; exploration constant = 1; global pheromone decay rate = 0.9; local pheromone decay rate = 0.5; pheromone sensitivity = 1; visibility sensitivity = 5;	100
ES	$\lambda = 10$, $\sigma = 1$, have been recommended	100
GA	Roulette wheel selection, single point crossover with a crossover probability of 1, and a mutation probability of 0.01.	100
PSO	$\omega = 0.6$, $c_1 = c_2 = 2$	100
GSK	P = 0.1, $k_f = 0.5$, $k_r = 0.9$, K = 10	100

2. 50% rank based for each problem in each dimension as follows:

$$\mathrm{SR} = 0.1 \times \sum_{i=1}^{29} rank_{10D} + 0.2 \times \sum_{i=1}^{29} rank_{30D} + 0.3 \times \sum_{i=1}^{29} rank_{50D} + 0.4 \times \sum_{i=1}^{29} rank_{100D}.$$

$$\text{Score 2} = \left(1 - \frac{SR - SR_{min}}{SR}\right) \times 50.$$

3. Combine the above two parts to find the final score as follows:

$$\text{Score} = \text{ Score 1} + \text{Score 2}.$$

Note that $f2$ has been excluded because it shows unstable behavior especially for higher dimensions.

Besides, to compare and analyze the solution quality from a statistical angle of different algorithms and to check the behavior of the stochastic algorithms [142], the results are compared using non-parametric statistical hypothesis tests: multi-problem Wilcoxon signed-rank test (to check the differences between all algorithms for all functions); at a 0.05 significance level, where R^+ denotes the sum of ranks for the test problems in which the first algorithm performs better than the second algorithm (in the first column), and R^- represents the sum of ranks for the test problems in which the first algorithm performs worse than the second algorithm (in the first column). Larger ranks indicate larger performance discrepancy. As a null hypothesis, it is assumed that there is no significance difference between the mean results of the two samples. Whereas the alternative hypothesis is that there is significance in the mean results of the two samples. For CEC2017, the number of test problems is N = 29 for D = 10, 30, 50 and 100 dimensions and 5% significance level. For CEC2011, the number of test problems is 22 and 5% significance level. Use the p value and compare it with the significance level. Reject the null hypothesis if the p-value is less

Table 7 Result of GSK in 10D

Function	Best	Median	Mean	Worst	SD
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
5	1.53E+01	2.01E+01	2.03E+01	2.58E+01	2.79E+00
6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
7	2.46E+01	3.10E+01	3.07E+01	3.74E+01	3.08E+00
8	1.45E+01	1.99E+01	2.02E+01	2.58E+01	2.92E+00
9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
10	7.54E+02	1.10E+03	1.06E+03	1.29E+03	1.33E+02
11	0.00E+00	0.00E+00	0.00E+00	1.02E−08	0.00E+00
12	1.42E+01	7.57E+01	8.93E+01	2.69E+02	7.26E+01
13	9.19E−01	6.56E+00	6.56E+00	9.73E+00	1.41E+00
14	5.19E−03	5.42E+00	5.88E+00	1.19E+01	3.06E+00
15	2.96E−03	1.75E−01	2.22E−01	5.00E−01	2.13E−01
16	3.29E−01	9.48E−01	4.27E+00	1.18E+01	4.95E+00
17	9.76E−01	9.94E+00	1.17E+01	2.52E+01	7.09E+00
18	2.05E−02	4.30E−01	3.20E−01	5.01E−01	1.92E−01
19	2.81E−02	6.95E−02	1.55E−01	1.80E+00	3.51E−01
20	3.12E−01	3.12E−01	1.19E+00	2.03E+01	3.99E+00
21	1.00E+02	2.20E+02	1.93E+02	2.31E+02	5.07E+01
22	1.00E+02	1.00E+02	1.00E+02	1.02E+02	4.87E−01
23	3.11E+02	3.18E+02	3.18E+02	3.29E+02	3.99E+00
24	2.78E+02	3.50E+02	3.44E+02	3.57E+02	1.87E+01
25	3.98E+02	4.34E+02	4.27E+02	4.46E+02	2.05E+01
26	3.00E+02	3.00E+02	3.00E+02	3.00E+02	0.00E+00
27	3.89E+02	3.90E+02	3.89E+02	3.90E+02	2.17E−01
28	3.00E+02	3.00E+02	3.12E+02	3.97E+02	3.20E+01
29	2.39E+02	2.48E+02	2.48E+02	2.57E+02	4.76E+00
30	3.96E+02	4.65E+02	4.56E+02	5.02E+02	3.44E+01

Table 8 Result of GSK in 30D

Function	Best	Median	Mean	Worst	SD
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
3	0.00E+00	5.66E−08	7.07E−07	8.54E−06	1.82E−06
4	9.28E−03	4.00E+00	1.11E+01	7.25E+01	2.17E+01
5	1.36E+02	1.62E+02	1.60E+02	1.73E+02	8.87E+00
6	0.00E+00	5.47E−07	1.52E−06	8.42E−06	2.36E−06
7	1.64E+02	1.87E+02	1.87E+02	1.99E+02	8.40E+00
8	1.24E+02	1.56E+02	1.55E+02	1.73E+02	1.11E+01
9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
10	5.84E+03	6.68E+03	6.69E+03	7.16E+03	3.54E+02
11	6.00E−01	8.57E+00	3.30E+01	9.85E+01	3.83E+01
12	1.03E+03	5.32E+03	6.62E+03	1.91E+04	4.59E+03
13	4.61E+01	9.11E+01	9.83E+01	1.82E+02	3.42E+01
14	4.72E+01	5.73E+01	5.69E+01	6.65E+01	5.49E+00
15	2.13E+00	9.17E+00	1.45E+01	7.48E+01	1.49E+01
16	4.35E+02	7.76E+02	7.96E+02	1.15E+03	1.94E+02
17	6.28E+01	2.04E+02	1.89E+02	3.77E+02	9.44E+01
18	2.25E+01	3.69E+01	3.68E+01	4.80E+01	5.42E+00
19	3.42E+00	1.13E+01	1.29E+01	2.22E+01	6.02E+00
20	1.34E+00	5.51E+01	1.08E+02	4.51E+02	1.14E+02
21	3.20E+02	3.49E+02	3.46E+02	3.55E+02	8.30E+00
22	1.00E+02	1.00E+02	1.00E+02	1.00E+02	0.00E+00
23	3.50E+02	4.86E+02	4.70E+02	5.07E+02	4.49E+01
24	5.29E+02	5.70E+02	5.68E+02	5.86E+02	1.45E+01
25	3.87E+02	3.87E+02	3.87E+02	3.87E+02	2.11E−01
26	6.93E+02	9.56E+02	9.87E+02	2.12E+03	2.49E+02
27	4.80E+02	4.92E+02	4.93E+02	5.14E+02	8.02E+00
28	3.00E+02	3.00E+02	3.21E+02	4.04E+02	4.22E+01
29	4.23E+02	5.92E+02	5.77E+02	7.69E+02	1.01E+02
30	1.94E+03	2.09E+03	2.08E+03	2.36E+03	9.27E+01

Table 9 Result of GSK in 50*D*

Function	Best	Median	Mean	Worst	SD
1	1.71E+01	4.73E+02	1.09E+03	4.61E+03	1.24E+03
3	1.61E+03	3.79E+03	3.85E+03	6.73E+03	1.51E+03
4	1.33E−02	7.21E+01	8.33E+01	1.46E+02	5.00E+01
5	2.65E+02	3.25E+02	3.20E+02	3.45E+02	1.79E+01
6	3.11E−07	2.80E−06	3.78E−06	1.64E−05	3.52E−06
7	3.29E+02	3.74E+02	3.70E+02	3.86E+02	1.41E+01
8	2.90E+02	3.28E+02	3.24E+02	3.38E+02	1.36E+01
9	0.00E+00	0.00E+00	1.07E−02	8.95E−02	2.79E−02
10	1.21E+04	1.28E+04	1.30E+04	1.37E+04	4.50E+02
11	2.34E+01	2.96E+01	3.45E+01	1.45E+02	2.32E+01
12	2.16E+03	7.71E+03	9.46E+03	3.17E+04	7.01E+03
13	7.41E+01	6.24E+02	1.49E+03	8.98E+03	2.16E+03
14	5.76E+01	1.28E+02	1.24E+02	1.42E+02	1.87E+01
15	2.52E+01	3.62E+01	4.20E+01	1.00E+02	1.68E+01
16	1.30E+02	2.01E+03	1.83E+03	2.70E+03	6.59E+02
17	7.75E+02	1.39E+03	1.35E+03	1.63E+03	1.90E+02
18	1.78E+02	5.01E+02	5.98E+02	1.45E+03	3.37E+02
19	1.84E+01	2.92E+01	3.05E+01	5.13E+01	9.59E+00
20	1.17E+03	1.40E+03	1.37E+03	1.62E+03	1.28E+02
21	4.90E+02	5.25E+02	5.21E+02	5.46E+02	1.31E+01
22	1.00E+02	1.31E+04	1.10E+04	1.35E+04	4.85E+03
23	4.20E+02	4.46E+02	5.42E+02	7.49E+02	1.39E+02
24	5.01E+02	5.24E+02	6.34E+02	8.11E+02	1.39E+02
25	4.60E+02	5.64E+02	5.56E+02	6.11E+02	4.62E+01
26	1.06E+03	1.29E+03	1.27E+03	1.42E+03	9.18E+01
27	5.18E+02	5.64E+02	5.92E+02	9.16E+02	8.29E+01
28	4.59E+02	4.97E+02	4.94E+02	5.70E+02	2.24E+01
29	3.28E+02	3.55E+02	3.60E+02	4.09E+02	2.23E+01
30	5.79E+05	5.81E+05	5.96E+05	6.52E+05	2.24E+04

than or equal the significance level (5%). All the p values in this paper were computed using SPSS (version 20.00).

3.3 Experimental results and discussions

3.3.1 Results of the proposed approach (GSK) on CEC2017

The statistical results of the GSK on the CEC2017 benchmarks with 10, 30, 50 and 100 dimensions are summarized in Tables 7, 8, 9 and 10, respectively. It includes the obtained best, median, mean, worst values and the standard deviations of error from optimum solution of the proposed GSK over 51 runs for all 29 benchmark functions.

Generally, from Tables 7, 8, 9 and 10 it can be clearly seen that GSK succeeded at solving, at least once, six problems in 10D, four problems in 30D, three problems in 50D, and one problem in 100D. GSK has outstanding performance on unimodal problems (*f1–f3*), GSK is able to find the global optimal solution consistently over 51 runs in 10 and 30 dimensions. In 50 dimensions, the optimum is detected in 1 case, while the mean error ranges from 1.09E+03 to 3.85E+03 and the standard deviation ranges from 1.24E+03 to 1.51E+03. In 100 dimensions, the mean error ranges from 5.80E+03 to 1.15E+05 and the standard deviation ranges from 4.63E+03 to 2.15E+04.

As for the simple multimodal functions (*f4–f10*), in 10 dimensions the optimum is detected continuously over 51 runs in 3 cases, while the mean error ranges from 0.00E+00 to 1.06E+03 and the standard deviation ranges from 0.00E+00 to 1.33E+02. In 30 dimensions, the optimum is detected continuously over 51 runs in 2 cases, while the mean error ranges from 0.00E+00 to 6.69E+03 and the standard deviation ranges from 0.00E+00 to 3.54E+02. In 50 dimensions, the optimum is detected in 2 case, while the mean error ranges from 3.78E−06 to 1.30E+04 and the standard deviation ranges from 3.52E−06 to 4.52E+02. In 100 dimensions, the mean error ranges from 1.72E−03 to 2.95E+04 and the standard deviation ranges from 6.46E−03

Table 10 Result of GSK in 100*D*

Function	Best	Median	Mean	Worst	SD
1	3.96E−01	5.38E+03	5.80E+03	2.11E+04	4.63E+03
3	6.01E+04	1.15E+05	1.15E+05	1.48E+05	2.15E+04
4	8.46E+01	2.17E+02	2.05E+02	2.89E+02	4.57E+01
5	7.26E+01	7.70E+02	5.31E+02	8.14E+02	3.39E+02
6	1.25E−05	6.83E−05	1.72E−03	3.14E−02	6.46E−03
7	8.33E+02	8.76E+02	8.75E+02	9.16E+02	1.83E+01
8	5.90E+01	7.36E+02	4.92E+02	8.18E+02	3.41E+02
9	5.45E+00	7.69E+00	8.46E+00	1.72E+01	3.41E+00
10	2.87E+04	2.94E+04	2.95E+04	3.04E+04	4.44E+02
11	1.61E+02	2.65E+02	2.78E+02	4.64E+02	6.85E+01
12	2.25E+04	6.53E+04	8.34E+04	3.23E+05	7.52E+04
13	5.17E+01	2.85E+03	3.20E+03	9.31E+03	2.64E+03
14	3.30E+02	2.53E+03	4.64E+03	1.69E+04	4.47E+03
15	3.20E+01	4.23E+02	7.33E+02	5.20E+03	1.09E+03
16	2.87E+02	8.25E+02	2.27E+03	7.04E+03	2.61E+03
17	2.14E+03	4.12E+03	3.91E+03	4.59E+03	6.68E+02
18	1.93E+04	4.43E+04	5.73E+04	1.87E+05	3.63E+04
19	5.04E+01	8.41E+02	1.00E+03	3.25E+03	8.30E+02
20	3.97E+03	4.52E+03	4.46E+03	4.80E+03	2.22E+02
21	2.83E+02	3.29E+02	6.07E+02	1.02E+03	3.37E+02
22	2.86E+04	3.01E+04	3.00E+04	3.09E+04	4.57E+02
23	5.86E+02	6.11E+02	6.11E+02	6.49E+02	1.58E+01
24	8.89E+02	9.33E+02	9.32E+02	9.69E+02	1.70E+01
25	7.61E+02	8.20E+02	8.21E+02	8.99E+02	4.34E+01
26	3.33E+03	3.63E+03	3.66E+03	3.96E+03	1.78E+02
27	6.20E+02	6.46E+02	6.57E+02	7.30E+02	3.07E+01
28	4.99E+02	5.57E+02	5.53E+02	6.09E+02	3.25E+01
29	9.12E+02	1.21E+03	1.21E+03	1.56E+03	1.74E+02
30	2.41E+03	3.02E+03	2.99E+03	3.72E+03	2.73E+02

to 4.44E + 02. Considering hybrid functions (*f11–f20*), GSK is able to obtain good solutions for all test problems, except for f10 and f12 in which the performance was reduced when D increased.

Finally, in regards to composition functions (*f21–f30*),which considered the most difficult problems in the benchmark suite as they are highly multimodal, non-separable and possess different properties around huge number of local optima [140].the optimum is detected in 1 case, while the mean error ranges from 1.00E + 02 to 4.56E + 02, from 1.00E + 02 to 2.08E + 03, from 3.60E + 02 to 5.96E + 05, and from 5.53E + 02to 3.00E + 04, while the standard deviation ranges from 0.00E + 00 to 5.07E + 01, from 0.00E + 00 to 2.49E + 02, from 1.31E + 01 to 2.24E + 04, and from 1.58E + 01 to 4.57E + 02 in 10, 30, 50 and 100 dimensions, respectively. Thus, GSK is often trapped in local optimum which is still not far away from the optimum in all functions. Therefore, it can be concluded that in all functions for all the four dimensionalities, the differences between mean and median are small even in the cases when the final results are far away from the optimum, regardless of the dimensions. That implies the GSK is a robust algorithm. Finally, due to insignificant difference between the results in four dimensions, it can be concluded that the performance of the GSK algorithm slightly diminishes, and it is still more stable and robust against the curse of dimensionality i.e. it is overall steady as the dimensions of the problems increases.

3.3.2 Comparison against state-of-the-art algorithms

The statistical results of the comparisons on the benchmarks with 10, 30, 50 and 100 dimensions are summarized in Tables 11, 12, 13, 14, 15, 16, 17, and 18 respectively. It includes the obtained best and the standard deviations of error from optimum solution of GSK and other ten state-of-the-art algorithms over 51 runs for all 29 benchmark functions. The best results are marked in bold for all problems. Ranking of the algorithms using score metric on the CEC

Table 11 Experimental results of TLBO, SFS, DE, GA, ES and GSK over 51 independent runs on 29 test functions of 10 variables with 100,000 FES

Function	TLBO	SFS	DE	GA	ES	GSK
1	1.96E+03 ± 2.52E+03	5.60E+03 ± 3.26E+03	**0.00E+00 ± 0.00E+00**	1.14E+06 ± 2.18E+05	7.20E+09 ± 2.05E+09	**0.00E+00 ± 0.00E+00**
3	**0.00E+00 ± 0.00E+00**	2.05E−02 ± 1.17E−02	**0.00E+00 ± 0.00E+00**	1.24E+04 ± 7.87E+03	2.47E+04 ± 7.25E+03	**0.00E+00 ± 0.00E+00**
4	1.79E−01 ± 3.45E−01	9.37E−01 ± 6.74E−01	**0.00E+00 ± 0.00E+00**	1.17E+01 ± 3.29E+00	5.28E+02 ± 1.34E+02	**0.00E+00 ± 0.00E+00**
5	**8.24E+00 ± 3.38E+00**	8.68E+00 ± 3.30E+00	2.48E+01 ± 4.13E+00	4.90E+01 ± 1.22E+01	9.40E+01 ± 1.27E+01	2.03E+01 ± 2.79E+00
6	3.76E−02 ± 1.28E−01	5.35E−03 ± 1.59E−03	**0.00E+00 ± 0.00E+00**	1.24E+01 ± 4.54E+00	5.60E+01 ± 9.08E+00	**0.00E+00 ± 0.00E+00**
7	**1.76E+01 ± 3.25E+00**	2.33E+01 ± 3.60E+00	3.43E+01 ± 4.84E+00	6.87E+01 ± 1.46E+01	2.70E+02 ± 3.87E+01	3.07E+01 ± 3.08E+00
8	6.84E+00 ± 2.75E+00	**7.42E+00 ± 2.76E+00**	2.37E+01 ± 3.76E+00	3.54E+01 ± 8.94E+00	8.98E+01 ± 9.39E+00	2.02E+01 ± 2.92E+00
9	2.64E−01 ± 4.22E−01	6.68E−06 ± 4.34E−06	**0.00E+00 ± 0.00E+00**	2.39E+01 ± 1.75E+01	1.78E+03 ± 3.34E+02	**0.00E+00 ± 0.00E+00**
10	4.95E+02 ± 2.92E+02	**3.63E+02 ± 1.91E+02**	4.94E+02 ± 2.99E+02	7.52E+02 ± 2.36E+02	1.82E+03 ± 2.12E+02	1.06E+03 ± 1.33E+02
11	7.06E+00 ± 5.17E+00	4.61E+00 ± 1.25E+00	4.33E−02 ± 1.95E−01	8.21E+01 ± 7.89E+01	1.30E+03 ± 6.80E+02	**0.00E+00 ± 0.00E+00**
12	1.28E+04 ± 9.07E+03	5.04E+03 ± 2.11E+03	**8.06E+00 ± 1.89E+01**	1.12E+06 ± 1.78E+06	2.97E+08 ± 1.77E+08	8.93E+01 ± 7.26E+01
13	1.70E+03 ± 1.91E+03	4.55E+01 ± 9.83E+00	**6.56E+00 ± 1.86E+00**	2.62E+04 ± 2.19E+04	3.26E+06 ± 3.13E+06	**6.56E+00 ± 1.41E+00**
14	3.40E+01 ± 7.43E+00	2.20E+01 ± 3.82E+00	**4.30E−02 ± 1.96E−01**	1.48E+04 ± 6.43E+03	4.46E+03 ± 4.50E+03	5.88E+00 ± 3.06E+00
15	4.75E+01 ± 2.09E+01	1.00E+01 ± 2.14E+00	**3.60E−02 ± 1.03E−01**	1.38E+04 ± 1.26E+04	2.16E+04 ± 2.26E+04	2.22E−01 ± 2.13E−01
16	1.44E+01 ± 3.48E+01	4.17E+00 ± 3.16E+00	**2.93E+00 ± 4.65E+00**	3.00E+01 ± 4.18E+01	5.25E+02 ± 1.47E+02	4.27E+00 ± 4.95E+00
17	2.76E+01 ± 8.83E+00	2.34E+01 ± 5.66E+00	**4.82E+00 ± 6.77E+00**	4.12E+01 ± 1.29E+01	2.48E+02 ± 5.75E+01	1.17E+01 ± 7.09E+00
18	4.55E+03 ± 3.67E+03	5.22E+01 ± 1.05E+01	**8.18E−02 ± 1.73E−01**	8.30E+04 ± 9.59E+04	7.61E+06 ± 8.08E+06	3.20E−01 ± 1.92E−01
19	2.94E+01 ± 1.83E+01	5.84E+00 ± 8.94E−01	**6.27E−03 ± 1.11E−02**	6.30E+04 ± 4.21E+04	9.69E+04 ± 1.08E+05	1.55E−01 ± 3.51E−01
20	1.75E+01 ± 1.04E+01	1.21E+01 ± 3.31E+00	**1.10E−01 ± 1.63E−01**	1.02E+02 ± 5.01E+01	2.41E+02 ± 6.43E+01	1.19E+00 ± 3.99E+00
21	1.62E+02 ± 5.26E+01	**1.00E+02 ± 5.06E−02**	1.80E+02 ± 6.22E+01	1.54E+02 ± 1.49E+01	1.73E+02 ± 5.57E+00	1.93E+02 ± 5.07E+01
22	9.98E+01 ± 1.07E+01	**9.24E+01 ± 3.00E+01**	9.48E+01 ± 2.30E+01	1.25E+02 ± 2.17E+01	9.06E+02 ± 2.61E+02	1.00E+02 ± 4.87E−01
23	3.10E+02 ± 4.31E+00	**3.03E+02 ± 4.35E+01**	3.17E+02 ± 5.20E+00	3.63E+02 ± 1.51E+01	4.05E+02 ± 1.01E+01	3.18E+02 ± 3.99E+00
24	3.23E+02 ± 5.62E+01	**2.18E+02 ± 1.18E+02**	3.43E+02 ± 4.97E+01	3.90E+02 ± 3.40E+01	4.37E+02 ± 3.31E+01	3.44E+02 ± 1.87E+01
25	4.26E+02 ± 2.23E+01	4.21E+02 ± 2.30E+01	**4.11E+02 ± 2.10E+01**	4.30E+02 ± 2.52E+01	9.17E+02 ± 1.74E+02	4.27E+02 ± 2.05E+01
26	3.31E+02 ± 5.60E+01	2.92E+02 ± 4.40E+01	**3.00E+02 ± 0.00E+00**	4.19E+02 ± 8.88E+01	1.44E+03 ± 2.85E+02	**3.00E+02 ± 0.00E+00**
27	3.93E+02 ± 2.62E+00	3.92E+02 ± 1.85E+00	3.90E+02 ± 2.63E−01	4.10E+02 ± 9.10E+00	4.37E+02 ± 7.98E+00	**3.89E+02 ± 2.17E−01**
28	4.01E+02 ± 1.26E+02	**3.06E+02 ± 3.97E+01**	3.63E+02 ± 1.21E+02	5.82E+02 ± 1.56E+02	8.11E+02 ± 9.22E+01	3.12E+02 ± 3.20E+01
29	2.63E+02 ± 1.46E+01	2.59E+02 ± 1.18E+01	**2.40E+02 ± 4.73E+00**	3.04E+02 ± 4.09E+01	5.69E+02 ± 6.52E+01	2.48E+02 ± 4.76E+00
30	1.25E+05 ± 2.98E+05	2.03E+03 ± 1.63E+03	1.64E+04 ± 1.14E+05	3.73E+05 ± 3.78E+05	8.83E+05 ± 0.00E+00	**4.56E+02 ± 3.44E+01**

2017 functions is given in Table 19. The multi-problem Wilcoxon signed-rank GSK and others in 10D, 30D, 50D and 100D are summarized in Table 20.

Firstly, regarding evolutionary and physical based algorithms in four dimensions, it can be observed that SFS and DE algorithms can be good at different functions in some or all dimensions. However, GA and ES perform poorly on all the functions in all dimensions. Generally, GSK, SFS and DE do significantly better than the others on most functions in different dimensions.

On the other hand, regarding swarm intelligence-based algorithms, it can be obviously shown that AMO is competitive with GSK on some functions in different dimensions. However, GSK is superior to the others in most functions in all dimensions. Actually, it can be obviously seen that the performance of all compared algorithms shows complete and/or significant deterioration with the growth of the search-space dimensionality from 10D to 100D while the performance of the GSK algorithm slightly diminishes, and it is still more stable, efficient and robust against the curse of dimensionality.

Secondly, the performance of GSK and other competitive algorithms on the functions of different dimensions is discussed. Table 19 clearly shows that GSK gets the first ranking among all algorithms, followed by DE and AMO in second and third place, respectively. However, GWO, ACO and ES are the poorest algorithms, respectively. The score of all algorithms on the CEC 2017 functions is shown in Fig. 11. Table 20 summarizes the statistical analysis results of applying multiple-problem Wilcoxon's test between GSK and other compared algorithms for 10D, 30D, 50D and 100D problems.

From Table 20, we can see that GSK obtains higher R^+ values than R^- in all the cases with exception to DE and AMO in 10D. Precisely, we can draw the following conclusions: GSK outperforms TLBO, GA, ES, GWO, PSO, BBO and ACO significantly in all dimensions with exception to DE, SFS and AMO in 10, 30 and 50 dimensions and SFS and AMO 100 dimensions. Thus, according to the Wilcoxon's test at $\alpha = 0.05$, the significance difference can be observed in 28 cases out of 40, which means that GSK is

Table 12 Experimental results of GWO, AMO, PSO, BBO, ACO and GSK over 51 independent runs on 29 test functions of 10 variables with 100,000 FES

Function	GWO	AMO	PSO	BBO	ACO	GSK
1	1.50E+08 ± 6.50E+07	9.84E+00 ± 2.69E+01	2.30E+03 ± 3.05E+03	1.12E+06 ± 2.26E+05	1.57E+10 ± 3.75E+09	**0.00E+00 ± 0.00E+00**
3	5.28E+02 ± 7.98E+02	2.90E−08 ± 9.16E−08	**0.00E+00 ± 0.00E+00**	8.35E+02 ± 8.07E+02	6.32E+03 ± 1.74E+03	**0.00E+00 ± 0.00E+00**
4	1.84E+01 ± 1.62E+01	1.39E+00 ± 6.29E−01	2.82E+00 ± 1.21E+00	6.20E+00 ± 1.72E+00	1.83E+02 ± 4.26E+01	**0.00E+00 ± 0.00E+00**
5	3.01E+01 ± 4.85E+00	**6.36E+00 ± 1.49E+00**	1.59E+01 ± 7.08E+00	7.96E+00 ± 2.77E+00	6.86E+01 ± 8.53E+00	2.03E+01 ± 2.79E+00
6	7.81E+00 ± 1.09E+00	**0.00E+00 ± 0.00E+00**	8.27E−02 ± 3.36E−01	9.19E−01 ± 0.00E+00	3.57E+01 ± 5.40E+00	**0.00E+00 ± 0.00E+00**
7	4.35E+01 ± 5.14E+00	1.82E+01 ± 1.42E+00	**1.72E+01 ± 4.46E+00**	2.31E+01 ± 3.82E+00	1.23E+02 ± 1.43E+01	3.07E+01 ± 3.08E+00
8	2.38E+01 ± 4.44E+00	**6.84E+00 ± 1.49E+00**	1.23E+01 ± 5.33E+00	8.66E+00 ± 3.05E+00	4.85E+01 ± 6.66E+00	2.02E+01 ± 2.92E+00
9	1.10E+01 ± 3.79E+00	**0.00E+00 ± 0.00E+00**	0.00E+00 ± 0.00E+00	3.15E−01 ± 8.16E−02	5.56E+02 ± 1.53E+02	**0.00E+00 ± 0.00E+00**
10	8.53E+02 ± 2.50E+02	3.65E+02 ± 1.01E+02	6.30E+02 ± 2.64E+02	**2.57E+02 ± 1.55E+02**	1.45E+03 ± 1.34E+02	1.06E+03 ± 1.33E+02
11	4.03E+01 ± 9.91E+00	2.76E+00 ± 7.90E−01	1.10E+01 ± 7.27E+00	8.92E+00 ± 4.26E+00	1.96E+02 ± 6.99E+01	**0.00E+00 ± 0.00E+00**
12	2.58E+06 ± 3.10E+06	1.13E+04 ± 6.49E+03	1.33E+04 ± 1.24E+04	2.55E+05 ± 1.60E+05	9.79E+08 ± 9.01E+08	**8.93E+01 ± 7.26E+01**
13	1.16E+04 ± 8.11E+03	2.67E+01 ± 8.99E+00	6.45E+03 ± 5.72E+03	8.09E+04 ± 7.12E+04	7.02E+06 ± 2.36E+07	**6.56E+00 ± 1.41E+00**
14	5.91E+02 ± 1.21E+03	**3.83E+00 ± 1.67E+00**	4.57E+01 ± 1.93E+01	1.20E+04 ± 1.01E+04	4.33E+02 ± 1.10E+02	5.88E+00 ± 3.06E+00
15	8.04E+02 ± 1.12E+03	1.75E+00 ± 5.40E−01	5.62E+01 ± 5.89E+01	1.95E+04 ± 1.94E+04	3.74E+03 ± 2.37E+03	**2.22E−01 ± 2.13E−01**
16	8.52E+01 ± 9.41E+01	**1.13E+00 ± 2.64E−01**	2.05E+02 ± 1.19E+02	1.06E+02 ± 8.75E+01	2.65E+02 ± 5.71E+01	4.27E+00 ± 4.95E+00
17	5.42E+01 ± 8.45E+00	**4.92E+00 ± 2.53E+00**	4.56E+01 ± 2.30E+01	2.74E+01 ± 3.49E+01	1.14E+02 ± 2.31E+01	1.17E+01 ± 7.09E+00
18	3.68E+04 ± 2.11E+04	3.13E+01 ± 8.02E+00	5.10E+03 ± 5.76E+03	1.45E+05 ± 1.01E+05	6.67E+07 ± 1.65E+08	**3.20E−01 ± 1.92E−01**
19	1.75E+03 ± 3.81E+03	1.11E+00 ± 4.28E−01	9.57E+01 ± 2.95E+02	4.34E+04 ± 4.02E+04	1.24E+04 ± 1.31E+04	**1.55E−01 ± 3.51E−01**
20	7.76E+01 ± 3.83E+01	**6.19E−04 ± 1.74E−03**	5.51E+01 ± 5.42E+01	1.29E+01 ± 5.91E+00	1.20E+02 ± 2.77E+01	1.19E+00 ± 3.99E+00
21	2.03E+02 ± 4.93E+01	**1.37E+02 ± 5.02E+01**	1.79E+02 ± 5.65E+01	2.00E+02 ± 3.07E+01	**1.37E+02 ± 1.29E+01**	1.93E+02 ± 5.07E+01
22	1.25E+02 ± 6.03E+00	9.91E+01 ± 7.18E+00	**9.38E+01 ± 2.55E+01**	1.06E+02 ± 7.47E+00	2.57E+02 ± 6.14E+01	1.00E+02 ± 4.87E−01
23	3.33E+02 ± 3.86E+00	**3.08E+02 ± 1.61E+00**	3.28E+02 ± 1.24E+01	3.13E+02 ± 4.97E+00	3.78E+02 ± 9.61E+00	3.18E+02 ± 3.99E+00
24	3.63E+02 ± 4.56E+00	**2.85E+02 ± 8.09E+01**	3.24E+02 ± 8.33E+01	3.36E+02 ± 2.66E+01	2.93E+02 ± 3.63E+01	3.44E+02 ± 1.87E+01
25	4.42E+02 ± 1.56E+01	**4.18E+02 ± 2.23E+01**	4.25E+02 ± 2.29E+01	4.34E+02 ± 2.26E+01	5.76E+02 ± 4.23E+01	4.27E+02 ± 2.05E+01
26	4.09E+02 ± 1.47E+02	3.00E+02 ± 0.00E+00	**2.74E+02 ± 7.63E+01**	3.40E+02 ± 1.51E+02	7.47E+02 ± 8.18E+01	3.00E+02 ± 0.00E+00
27	3.96E+02 ± 1.15E+00	3.91E+02 ± 2.38E+00	4.03E+02 ± 1.97E+01	3.97E+02 ± 4.39E+00	4.40E+02 ± 8.57E+00	**3.89E+02 ± 2.17E−01**
28	5.39E+02 ± 9.99E+01	**2.99E+02 ± 3.99E+00**	4.54E+02 ± 1.57E+02	5.48E+02 ± 9.68E+01	5.85E+02 ± 4.64E+01	3.12E+02 ± 3.20E+01
29	2.94E+02 ± 3.04E+01	2.64E+02 ± 8.17E+00	3.05E+02 ± 4.50E+01	2.65E+02 ± 1.56E+01	3.90E+02 ± 3.41E+01	**2.48E+02 ± 4.76E+00**
30	4.84E+05 ± 7.31E+05	7.42E+03 ± 5.17E+03	2.00E+05 ± 3.79E+05	4.63E+05 ± 5.39E+05	2.23E+07 ± 2.32E+07	**4.56E+02 ± 3.44E+01**

significantly better than 7 algorithms out of 10 algorithms on 29 test functions at $\alpha = 0.05$.

Alternatively, to be more precise, it is obvious from Table 20 that GSK is inferior to, equal to, superior to other algorithms in 71, 14, 205 out of the total 290 cases in 10D, 55, 7, 228 out of the total 290 cases in 30D, 56, 1, 233 out of the total 290 cases in 50D, 51, 0, 239 out of the total 290 cases in 100D, respectively. In summary, GSK is inferior to, equal to, superior to other algorithms in 233, 22, 905 cases, respectively out of total 1160 cases.

Thus, it can be concluded that the performance of GSK is almost better than the performance of compared algorithms in 78% of all cases, respectively, and it is just outperformed by other compared algorithms in 20% of all problems in all dimensions. Furthermore, it can be obviously deduced from Fig. 12 that the superiority of the GSK algorithm against the compared algorithms increases as the dimensions of the problems increases from 10 to 100 dimensions.

From the above results, comparisons and discussion through this section, the proposed GSK algorithm is of better searching quality, efficiency and robustness for solving small, moderate and high dimensions unconstrained global optimization problems. It is clear that the proposed GSK algorithm perform well, and it has shown its outstanding superiority with separable, non-separable, unimodal, multimodal, hybrid and composition functions with shifts in dimensionality, rotation, multiplicative noise in fitness and composition of functions.

Consequently, its performance is not influenced by all these obstacles. Contrarily, it greatly keeps the balance the local optimization speed and the global optimization diversity in challenging optimization environment with invariant performance. Besides, its performance is superior and competitive with the performance of the state-of-the-art well-known algorithms. Finally, it can be concluded that the proposed junior and senior phases help to maintain effectively the balance between the global exploration and local exploitation abilities for searching process of the GSK. Besides, GSK is very simple and easy to implement and program in many programming languages.

Furthermore, in order to analyze the convergence behavior of GSK and other state-of-the-art algorithms, the

Table 13 Experimental results of TLBO, SFS, DE, GA, ES and GSK over 51 independent runs on 29 test functions of 30 variables with 300,000 FES

Function	TLBO	SFS	DE	GA	ES	GSK
1	3.36E+03 ± 3.19E+03	3.17E+03 ± 3.76E+03	**0.00E+00 ± 0.00E+00**	6.06E+06 ± 1.62E+06	8.32E+10 ± 1.11E+10	**0.00E+00 ± 0.00E+00**
3	1.03E−04 ± 2.38E−04	5.36E+01 ± 2.72E+01	1.36E+02 ± 1.36E+02	6.81E+04 ± 2.02E+04	1.95E+05 ± 3.27E+04	**7.07E−07 ± 1.82E−06**
4	5.63E+01 ± 3.51E+01	5.91E+01 ± 4.04E+01	5.92E+01 ± 1.81E+00	1.51E+02 ± 4.09E+01	2.45E+04 ± 6.13E+03	**1.11E+01 ± 2.17E+01**
5	9.06E+01 ± 2.00E+01	**6.76E+01 ± 1.34E+01**	1.79E+02 ± 1.27E+01	2.26E+02 ± 2.93E+01	5.41E+02 ± 3.34E+01	1.60E+02 ± 8.87E+00
6	8.45E+00 ± 4.18E+00	1.18E−02 ± 2.04E−02	0.00E+00 ± 0.00E+00	3.86E+01 ± 9.95E+00	1.11E+02 ± 6.94E+00	**1.52E−06 ± 2.36E−06**
7	1.48E+02 ± 3.18E+01	**1.01E+02 ± 2.19E+01**	2.12E+02 ± 9.99E+00	3.24E+02 ± 5.43E+01	2.09E+03 ± 1.89E+02	1.87E+02 ± 8.40E+00
8	**7.06E+01 ± 1.44E+01**	7.50E+01 ± 1.96E+01	1.80E+02 ± 1.14E+01	2.43E+02 ± 3.07E+01	4.88E+02 ± 2.69E+01	1.55E+02 ± 1.11E+01
9	2.07E+02 ± 1.29E+02	2.47E+01 ± 6.83E+01	**0.00E+00 ± 0.00E+00**	1.12E+03 ± 1.42E+03	2.24E+04 ± 1.93E+03	**0.00E+00 ± 0.00E+00**
10	6.01E+03 ± 1.17E+03	**2.35E+03 ± 5.20E+02**	6.61E+03 ± 4.36E+02	4.48E+03 ± 8.44E+02	7.41E+03 ± 2.37E+02	6.69E+03 ± 3.54E+02
11	1.36E+02 ± 4.75E+01	4.57E+01 ± 2.85E+01	7.49E+01 ± 3.10E+01	1.33E+03 ± 1.01E+03	1.65E+04 ± 4.77E+03	**3.30E+01 ± 3.83E+01**
12	4.44E+04 ± 6.48E+04	2.75E+04 ± 1.38E+04	7.55E+03 ± 7.17E+03	4.29E+06 ± 3.34E+06	1.32E+10 ± 2.92E+09	**6.62E+03 ± 4.59E+03**
13	1.40E+04 ± 1.41E+04	9.39E+02 ± 2.79E+02	**8.24E+01 ± 9.84E+00**	2.54E+06 ± 2.22E+06	8.70E+09 ± 3.21E+09	9.83E+01 ± 3.42E+01
14	2.99E+03 ± 2.58E+03	8.72E+01 ± 9.94E+00	6.33E+01 ± 4.72E+00	9.85E+05 ± 9.57E+05	5.46E+06 ± 3.38E+06	**5.69E+01 ± 5.49E+00**
15	5.17E+03 ± 6.89E+03	1.52E+02 ± 2.55E+01	3.91E+01 ± 5.79E+00	7.21E+05 ± 2.53E+05	1.56E+09 ± 7.25E+08	**1.45E+01 ± 1.49E+01**
16	5.67E+02 ± 2.18E+02	**4.41E+02 ± 1.58E+02**	7.78E+02 ± 4.11E+02	1.17E+03 ± 3.00E+02	4.10E+03 ± 4.79E+02	7.96E+02 ± 1.94E+02
17	2.00E+02 ± 9.15E+01	1.05E+02 ± 6.29E+01	**1.02E+02 ± 5.03E+01**	6.43E+02 ± 2.04E+02	2.33E+03 ± 4.24E+02	1.89E+02 ± 9.44E+01
18	2.05E+05 ± 1.40E+05	3.32E+02 ± 7.40E+01	3.83E+01 ± 4.10E+00	3.77E+06 ± 4.84E+06	8.61E+07 ± 3.76E+07	**3.68E+01 ± 5.42E+00**
19	5.57E+03 ± 6.13E+03	5.83E+01 ± 9.10E+00	1.89E+01 ± 5.75E+00	8.32E+05 ± 3.31E+05	2.09E+09 ± 7.48E+08	**1.29E+01 ± 6.02E+00**
20	2.20E+02 ± 7.33E+01	1.30E+02 ± 6.05E+01	**6.03E+01 ± 6.50E+01**	4.10E+02 ± 1.11E+02	1.24E+03 ± 1.42E+02	1.08E+02 ± 1.14E+02
21	2.70E+02 ± 1.83E+01	**2.56E+02 ± 2.71E+01**	3.66E+02 ± 1.34E+01	4.57E+02 ± 3.28E+01	6.91E+02 ± 3.09E+01	3.46E+02 ± 8.30E+00
22	2.08E+02 ± 7.55E+02	**1.00E+02 ± 5.50E−04**	1.81E+03 ± 2.95E+03	5.16E+03 ± 1.60E+03	7.96E+03 ± 2.24E+02	**1.00E+02 ± 0.00E+00**
23	4.42E+02 ± 2.62E+01	**4.08E+02 ± 1.35E+01**	5.26E+02 ± 9.51E+00	6.77E+02 ± 3.70E+01	1.10E+03 ± 5.19E+01	4.70E+02 ± 4.49E+01
24	4.97E+02 ± 2.37E+01	**4.93E+02 ± 2.27E+01**	5.96E+02 ± 7.08E+00	8.14E+02 ± 6.40E+01	1.03E+03 ± 4.56E+01	5.68E+02 ± 1.45E+01
25	4.08E+02 ± 2.28E+01	**3.86E+02 ± 2.68E+00**	3.87E+02 ± 2.19E−02	5.61E+02 ± 9.50E+01	9.40E+03 ± 1.72E+03	3.87E+02 ± 2.11E−01
26	2.10E+03 ± 1.08E+03	1.32E+03 ± 8.12E+02	2.58E+03 ± 2.61E+02	3.61E+03 ± 8.15E+02	9.14E+03 ± 6.85E+02	**9.87E+02 ± 2.49E+02**
27	5.34E+02 ± 2.02E+01	5.16E+02 ± 1.25E+01	4.96E+02 ± 7.01E+00	6.13E+02 ± 4.04E+01	8.91E+02 ± 5.91E+01	**4.93E+02 ± 8.02E+00**
28	3.85E+02 ± 5.63E+01	3.61E+02 ± 4.99E+01	**3.20E+02 ± 4.41E+01**	5.66E+02 ± 4.52E+01	5.67E+03 ± 5.95E+02	3.21E+02 ± 4.22E+01
29	8.37E+02 ± 1.80E+02	5.53E+02 ± 1.04E+02	**5.42E+02 ± 9.62E+01**	9.07E+02 ± 1.78E+02	2.94E+03 ± 1.10E+02	5.77E+02 ± 1.01E+02
30	5.24E+03 ± 2.76E+03	8.22E+03 ± 3.40E+03	**2.00E+03 ± 5.62E+01**	3.45E+05 ± 1.43E+05	1.19E+09 ± 4.18E+08	2.08E+03 ± 9.27E+01

convergence characteristics in terms of the best fitness value of the median run of all algorithms for some functions with dimensions 10,30, 50 and 100 is illustrated in the supplemental file (Fig. S1). It is clear that the convergence speed of the GSK algorithm is fast at the early stage of the optimization process for all functions with different shapes, complexity, and dimensions. Furthermore, the convergence speed is dramatically decreased, and its improvement is found to be significant in the middle and later stages of the optimization process.

Additionally, the convergent figure suggests that the GSK algorithm can reach the global solution or better solution in most problems in a fewer number of generations less than the maximum predetermined number of generations. In general, GSK is scalable enough and can balance greatly the exploration and exploitation abilities until the maximum FEs is reached. Therefore, the proposed GSK algorithm is proven to be an effective and powerful approach for solving unconstrained global optimization problems within limited number of function evaluations which is a very important issue when dealing with real-world problems. Finally, it can be obviously deduced from Fig. S1 that the GSK converges faster than other compared algorithms in most cases especially with high dimensions.

3.3.3 Algorithm complexity

The algorithm complexity of all algorithms on 10, 30, 50 dimensions are shown in Tables 21, 22, 23, respectively. All experiments were implemented and executed using MATLAB R2014a running on a PC with core i7-4790 (3.60 GHz) CPU and 12 GB RAM running win 10 OS. In order to evaluate the computational complexity of the compared algorithms we follow the guidelines described in [40].

T_0 is the time in seconds needed to run the following program:

Table 14 Experimental results of GWO, AMO, PSO, BBO, ACO and GSK over 51 independent runs on 29 test functions of 30 variables with 300,000 FES

Function	GWO	AMO	PSO	BBO	ACO	GSK
1	3.68E+09 ± 7.80E+08	5.34E+00 ± 7.57E+00	3.43E+03 ± 3.97E+03	4.20E+06 ± 1.87E+05	8.95E+10 ± 2.59E+10	**0.00E+00 ± 0.00E+00**
3	2.82E+04 ± 6.90E+03	4.56E+03 ± 1.48E+03	1.48E+02 ± 5.90E+01	4.39E+04 ± 2.58E+04	2.08E+07 ± 1.23E+08	**7.07E−07 ± 1.82E−06**
4	2.58E+02 ± 3.54E+01	**4.60E+00 ± 1.07E+01**	8.59E+01 ± 3.22E+01	9.99E+01 ± 2.27E+01	9.61E+03 ± 1.72E+03	1.11E+01 ± 2.17E+01
5	2.02E+02 ± 1.63E+01	5.41E+01 ± 6.68E+00	1.14E+02 ± 2.67E+01	**4.21E+01 ± 1.06E+01**	4.11E+02 ± 2.27E+01	1.60E+02 ± 8.87E+00
6	2.60E+01 ± 2.54E+00	**0.00E+00 ± 0.00E+00**	3.09E+00 ± 4.01E+00	8.98E−01 ± 4.07E−02	8.31E+01 ± 6.11E+00	1.52E−06 ± 2.36E−06
7	2.81E+02 ± 1.90E+01	**9.15E+01 ± 6.90E+00**	9.71E+01 ± 1.66E+01	1.14E+02 ± 1.46E+01	9.44E+02 ± 9.83E+01	1.87E+02 ± 8.40E+00
8	1.87E+02 ± 1.53E+01	**5.42E+01 ± 5.76E+00**	9.73E+01 ± 2.11E+01	4.31E+01 ± 1.00E+01	3.63E+02 ± 1.97E+01	1.55E+02 ± 1.11E+01
9	1.42E+03 ± 3.61E+02	1.05E−01 ± 2.07E−01	1.01E+03 ± 1.15E+03	1.09E+02 ± 8.42E+01	1.39E+04 ± 1.46E+03	**0.00E+00 ± 0.00E+00**
10	6.21E+03 ± 4.87E+02	**3.62E+03 ± 2.66E+02**	3.20E+03 ± 5.52E+02	2.29E+03 ± 4.48E+02	7.38E+03 ± 2.24E+02	6.69E+03 ± 3.54E+02
11	5.22E+02 ± 3.70E+02	4.91E+01 ± 2.47E+01	1.14E+02 ± 3.55E+01	1.43E+03 ± 1.40E+03	5.54E+03 ± 1.53E+03	**3.30E+01 ± 3.83E+01**
12	2.96E+08 ± 8.71E+07	6.07E+04 ± 6.88E+04	1.43E+05 ± 9.98E+04	3.52E+06 ± 2.19E+06	1.89E+10 ± 4.59E+09	**6.62E+03 ± 4.59E+03**
13	1.10E+08 ± 5.32E+07	6.78E+03 ± 3.23E+03	1.49E+04 ± 1.63E+04	1.51E+06 ± 4.77E+05	1.23E+10 ± 7.17E+09	**9.83E+01 ± 3.42E+01**
14	1.31E+05 ± 2.24E+05	2.22E+03 ± 1.39E+03	1.05E+04 ± 8.32E+03	8.23E+05 ± 8.54E+05	9.44E+06 ± 2.25E+07	**5.69E+01 ± 5.49E+00**
15	2.94E+06 ± 5.30E+06	4.22E+02 ± 5.61E+02	7.96E+03 ± 8.62E+03	7.01E+05 ± 3.26E+05	3.56E+09 ± 2.14E+09	**1.45E+01 ± 1.49E+01**
16	1.25E+03 ± 2.51E+02	**5.52E+02 ± 1.19E+02**	8.59E+02 ± 2.38E+02	8.95E+02 ± 3.11E+02	2.90E+03 ± 2.10E+02	7.96E+02 ± 1.94E+02
17	3.96E+02 ± 1.36E+02	**8.49E+01 ± 1.77E+01**	3.57E+02 ± 1.64E+02	3.81E+02 ± 2.16E+02	1.28E+03 ± 1.66E+02	1.89E+02 ± 9.44E+01
18	1.07E+06 ± 7.22E+05	1.43E+05 ± 5.21E+04	1.84E+05 ± 1.36E+05	1.95E+06 ± 1.96E+06	1.78E+08 ± 1.13E+08	**3.68E+01 ± 5.42E+00**
19	5.71E+06 ± 2.88E+06	1.32E+03 ± 1.53E+03	7.77E+03 ± 1.12E+04	8.37E+05 ± 3.40E+05	3.91E+09 ± 2.60E+09	**1.29E+01 ± 6.02E+00**
20	4.57E+02 ± 1.33E+02	1.42E+02 ± 4.76E+01	3.78E+02 ± 1.36E+02	4.33E+02 ± 1.85E+02	8.37E+02 ± 1.02E+02	**1.08E+02 ± 1.14E+02**
21	3.83E+02 ± 1.68E+01	**2.53E+02 ± 6.95E+00**	3.06E+02 ± 2.35E+01	2.49E+02 ± 1.04E+01	5.90E+02 ± 1.72E+01	3.46E+02 ± 8.30E+00
22	3.91E+03 ± 2.80E+03	**1.00E+02 ± 0.00E+00**	9.62E+02 ± 1.61E+03	1.58E+03 ± 1.49E+03	5.74E+03 ± 4.29E+02	**1.00E+02 ± 0.00E+00**
23	5.67E+02 ± 1.89E+01	**3.98E+02 ± 9.02E+00**	5.08E+02 ± 5.60E+01	4.02E+02 ± 1.24E+01	9.40E+02 ± 4.40E+01	4.70E+02 ± 4.49E+01
24	6.33E+02 ± 1.27E+01	**4.64E+02 ± 8.44E+00**	5.73E+02 ± 6.32E+01	4.67E+02 ± 1.33E+01	1.06E+03 ± 5.58E+01	5.68E+02 ± 1.45E+01
25	5.02E+02 ± 3.08E+01	**3.87E+02 ± 1.07E+00**	3.90E+02 ± 8.76E+00	3.93E+02 ± 9.81E+00	3.49E+03 ± 6.51E+02	**3.87E+02 ± 2.11E−01**
26	3.10E+03 ± 1.47E+02	1.49E+03 ± 1.95E+02	1.27E+03 ± 1.39E+03	1.63E+03 ± 1.56E+02	7.10E+03 ± 4.27E+02	**9.87E+02 ± 2.49E+02**
27	5.63E+02 ± 2.37E+01	5.16E+02 ± 4.81E+00	5.43E+02 ± 3.02E+01	5.26E+02 ± 7.66E+00	1.14E+03 ± 7.37E+01	**4.93E+02 ± 8.02E+00**
28	6.63E+02 ± 5.98E+01	**3.14E+02 ± 3.61E+01**	4.14E+02 ± 2.58E+01	4.33E+02 ± 2.47E+01	3.51E+03 ± 4.77E+02	3.21E+02 ± 4.22E+01
29	1.08E+03 ± 1.62E+02	**5.33E+02 ± 2.46E+01**	7.56E+02 ± 1.79E+02	7.18E+02 ± 1.45E+02	2.59E+03 ± 2.20E+02	5.77E+02 ± 1.01E+02
30	2.50E+07 ± 8.05E+06	4.71E+03 ± 7.99E+02	5.95E+03 ± 2.82E+03	3.31E+05 ± 1.23E+05	3.16E+09 ± 9.12E+08	**2.08E+03 ± 9.27E+01**

```
for i=1:1000000
    x=x + x;
    x=x/2;
    x=x*x;
    x=sqrt(x);
    x=log(x);
    x=exp(x);
    x=x/(x+2);
end
```

T_1 is the time in seconds to execute 200,000 evaluations of benchmark function *f18* by itself with D dimensions, and T_2 is the mean time of executing the compared algorithms 5 times for 200,000 valuations for *f18* in D dimensions. Finally, the algorithm complexity is shown in T_2, T_1, and $(T_2 - T_1)/T_0$.

Actually, it can be easily noticed from these tables that the complexity of GSK algorithm should be considered with some care, since the measured values of run-time are very small relative to the remaining algorithms. Thus, regarding implementation and design of algorithms, it can be derived that it is easily implemented more than compared algorithms.

3.3.4 Results of the proposed approach (GSK) on CEC2011

The statistical results of the GSK on the CEC2011 benchmarks are summarized in Table 24, respectively. It includes the obtained best, median, mean, worst values and the standard deviations of objective function value of the proposed GSK over 25 runs for all 22 benchmark functions. Generally, from Table 24 it can be clearly seen that GSK is able to find the global optimal solution consistently in 3 test functions over 25 runs. With respect to first test function, although the optimal solutions are not consistently found, the best result achieved is very close to the global optimal solution which can be verified by the very small standard deviation. Additionally, regarding the remaining test functions, the

Table 15 Experimental results of TLBO, SFS, DE, GA, ES and GSK over 51 independent runs on 29 test functions of 50 variables with 500,000 FES

Function	TLBO	SFS	DE	GA	ES	GSK
1	2.56E+03 ± 3.07E+03	3.65E+03 ± 5.38E+03	**7.43E−01 ± 1.81E+00**	1.93E+07 ± 4.99E+06	2.01E+11 ± 1.43E+10	1.09E+03 ± 1.24E+03
3	**1.40E+03 ± 1.03E+03**	4.44E+03 ± 1.28E+03	9.29E+04 ± 1.68E+04	8.00E+04 ± 1.78E+04	3.90E+05 ± 5.44E+04	3.85E+03 ± 1.51E+03
4	1.03E+02 ± 4.48E+01	1.10E+02 ± 5.13E+01	**8.03E+01 ± 4.94E+01**	3.03E+02 ± 8.39E+01	6.57E+04 ± 1.15E+04	8.33E+01 ± 5.00E+01
5	**1.87E+02 ± 3.14E+01**	2.13E+02 ± 4.68E+01	3.50E+02 ± 1.40E+01	4.47E+02 ± 5.07E+01	1.01E+03 ± 4.69E+01	3.20E+02 ± 1.79E+01
6	2.17E+01 ± 5.12E+00	1.01E−01 ± 1.99E−01	**2.60E−07 ± 1.09E−06**	4.11E+01 ± 4.95E+00	1.28E+02 ± 6.97E+00	3.78E−06 ± 3.52E−06
7	3.84E+02 ± 6.37E+01	**2.56E+02 ± 5.48E+01**	4.07E+02 ± 1.09E+01	5.87E+02 ± 6.93E+01	4.46E+03 ± 2.60E+02	3.70E+02 ± 1.41E+01
8	**2.02E+02 ± 2.96E+01**	2.03E+02 ± 4.35E+01	3.53E+02 ± 1.42E+01	4.53E+02 ± 5.51E+01	1.02E+03 ± 4.95E+01	3.24E+02 ± 1.36E+01
9	2.68E+03 ± 1.47E+03	2.90E+02 ± 4.52E+02	3.99E−02 ± 1.26E−01	3.96E+03 ± 2.17E+03	6.83E+04 ± 6.85E+03	**1.07E−02 ± 2.79E−02**
10	1.02E+04 ± 2.67E+03	**4.68E+03 ± 6.67E+02**	1.30E+04 ± 7.47E+02	8.76E+03 ± 7.41E+02	1.38E+04 ± 3.75E+02	1.30E+04 ± 4.50E+02
11	2.16E+02 ± 6.69E+01	1.28E+02 ± 3.60E+01	1.43E+02 ± 2.27E+01	6.84E+03 ± 4.28E+03	4.70E+04 ± 9.97E+03	**3.45E+01 ± 2.32E+01**
12	5.90E+05 ± 1.31E+06	3.19E+05 ± 2.42E+05	6.19E+04 ± 3.75E+04	1.65E+07 ± 8.55E+06	8.68E+10 ± 1.41E+10	**9.46E+03 ± 7.01E+03**
13	5.23E+03 ± 3.91E+03	3.50E+03 ± 3.20E+03	**5.33E+02 ± 1.39E+03**	2.17E+06 ± 5.56E+05	4.08E+10 ± 7.67E+09	1.49E+03 ± 2.16E+03
14	4.92E+04 ± 4.29E+04	1.68E+02 ± 1.53E+01	1.25E+02 ± 9.35E+00	2.22E+06 ± 1.10E+06	4.58E+07 ± 1.66E+07	**1.24E+02 ± 1.87E+01**
15	6.65E+03 ± 5.53E+03	3.00E+02 ± 4.76E+01	1.08E+02 ± 1.00E+01	1.59E+06 ± 3.13E+05	1.42E+10 ± 3.99E+09	**4.20E+01 ± 1.68E+01**
16	1.17E+03 ± 3.09E+02	**1.18E+03 ± 3.37E+02**	2.51E+03 ± 6.16E+02	2.67E+03 ± 5.68E+02	8.06E+03 ± 7.76E+02	1.83E+03 ± 6.59E+02
17	9.96E+02 ± 2.23E+02	**7.49E+02 ± 1.96E+02**	1.19E+03 ± 4.77E+02	1.26E+03 ± 3.48E+02	5.07E+04 ± 4.10E+04	1.35E+03 ± 1.90E+02
18	5.45E+05 ± 3.28E+05	**4.53E+02 ± 1.55E+02**	7.67E+02 ± 1.31E+03	2.72E+06 ± 2.05E+06	2.27E+08 ± 8.65E+07	5.98E+02 ± 3.37E+02
19	1.32E+04 ± 7.59E+03	1.22E+02 ± 2.92E+01	6.24E+01 ± 6.16E+00	5.69E+05 ± 2.18E+05	5.22E+09 ± 1.46E+09	**3.05E+01 ± 9.59E+00**
20	5.81E+02 ± 2.76E+02	**5.01E+02 ± 2.20E+02**	9.98E+02 ± 5.60E+02	1.52E+03 ± 2.68E+02	2.76E+03 ± 1.81E+02	1.37E+03 ± 1.28E+02
21	3.91E+02 ± 3.70E+01	**3.47E+02 ± 3.36E+01**	5.52E+02 ± 1.28E+01	7.20E+02 ± 6.23E+01	1.24E+03 ± 6.41E+01	5.21E+02 ± 1.31E+01
22	7.62E+03 ± 5.62E+03	**2.44E+03 ± 2.74E+03**	1.32E+04 ± 3.25E+02	1.01E+04 ± 7.97E+02	1.44E+04 ± 3.51E+02	1.10E+04 ± 4.85E+03
23	7.01E+02 ± 6.69E+01	5.98E+02 ± 4.60E+01	7.69E+02 ± 2.12E+01	9.94E+02 ± 4.44E+01	1.99E+03 ± 7.95E+01	**5.42E+02 ± 1.39E+02**
24	7.25E+02 ± 5.00E+01	6.91E+02 ± 3.95E+01	8.48E+02 ± 1.38E+01	1.27E+03 ± 9.86E+01	1.76E+03 ± 5.64E+01	**6.34E+02 ± 1.39E+02**
25	5.71E+02 ± 3.42E+01	5.60E+02 ± 2.79E+01	**4.96E+02 ± 3.14E+01**	6.90E+02 ± 6.08E+01	3.90E+04 ± 5.17E+03	5.56E+02 ± 4.62E+01
26	5.28E+03 ± 2.03E+03	1.60E+03 ± 2.23E+03	4.40E+03 ± 1.82E+02	5.46E+03 ± 5.13E+02	1.86E+04 ± 1.02E+03	**1.27E+03 ± 9.18E+01**
27	8.98E+02 ± 1.36E+02	6.29E+02 ± 5.61E+01	**5.45E+02 ± 4.05E+01**	1.17E+03 ± 1.25E+02	1.86E+03 ± 1.42E+02	5.92E+02 ± 8.29E+01
28	5.04E+02 ± 2.40E+01	5.08E+02 ± 3.05E+01	**4.67E+02 ± 1.85E+01**	1.40E+03 ± 4.65E+02	1.23E+04 ± 8.03E+02	4.94E+02 ± 2.24E+01
29	1.54E+03 ± 3.86E+02	8.61E+02 ± 2.33E+02	1.18E+03 ± 5.28E+02	1.53E+03 ± 3.29E+02	1.69E+04 ± 1.99E+03	**3.60E+02 ± 2.23E+01**
30	9.35E+05 ± 1.49E+05	9.88E+05 ± 1.73E+05	**5.91E+05 ± 2.40E+04**	3.09E+06 ± 3.38E+06	8.50E+09 ± 1.80E+09	5.96E+05 ± 2.24E+04

differences between mean and median are small even in the cases when the final results are far away from the optimum, regardless of the dimensions. That implies the GSK is a robust algorithm.

Finally, due to the exact optimum of most of these problems are not available, it is difficult to gauge the absolute performance of the algorithm at this stage. However, relative performance will be evaluated in next subsection when the performance of GSK will be compared with the performance of other algorithms.

3.3.5 Comparison against state-of-the-art algorithms

The statistical results of the comparisons on the benchmarks are summarized in Tables 25 and 26, respectively. It includes the obtained best and the standard deviations of the objective function value of GSK and other ten state-of-the-art algorithms over 25 runs for all 22 benchmark functions. The best results are marked in bold and the second best results are underlined for all problems. Ranking of the algorithms using Friedman test [142] is given in Table 27, where the algorithm with smallest ranking is the better. The null hypothesis for this test is that "there is no difference among the performance of all algorithms" and that alternative hypothesis states "there is a difference among the performance of all algorithms". The multi-problem Wilcoxon signed-rank GSK and others are summarized in Table 28.

Firstly, regarding evolutionary and physical based algorithms in four dimensions, it can be observed that SFS and TLBO algorithms can be good at different functions and DE shows moderate performance compared with others. However, GA and ES perform poorly on most of the functions. Generally, GSK, SFS and TLBO do significantly better than the others on most functions.

On the other hand, regarding swarm intelligence-based algorithms, it can be obviously shown that AMO and BBO are competitive with GSK on some functions. However, GSK is superior to the others in most functions. Actually, it can be obviously seen that the performance of all compared algorithms shows complete and/or significant deterioration on the problems with the growth of the search-space dimensionality. Besides, GWO shows moderate performance

Table 16 Experimental results of GWO, AMO, PSO, BBO, ACO and GSK over 51 independent runs on 29 test functions of 50 variables with 500,000 FES

Function	GWO	AMO	PSO	BBO	ACO	GSK
1	1.27E+10 ± 2.94E+09	1.30E+03 ± 1.27E+03	3.87E+03 ± 6.33E+03	6.10E+06 ± 7.65E+05	1.94E+11 ± 5.32E+10	**1.09E+03 ± 1.24E+03**
3	7.54E+04 ± 1.25E+04	3.15E+04 ± 4.71E+03	**1.90E+03 ± 3.35E+02**	1.23E+05 ± 4.22E+04	1.31E+06 ± 4.92E+06	3.85E+03 ± 1.51E+03
4	9.59E+02 ± 2.18E+02	**7.53E+01 ± 4.77E+01**	1.57E+02 ± 4.67E+01	1.43E+02 ± 5.04E+01	3.28E+04 ± 3.36E+03	8.33E+01 ± 5.00E+01
5	4.22E+02 ± 2.51E+01	1.34E+02 ± 1.30E+01	2.31E+02 ± 4.15E+01	**8.37E+01 ± 1.39E+01**	8.46E+02 ± 2.80E+01	3.20E+02 ± 1.79E+01
6	3.84E+01 ± 4.11E+00	2.10E−04 ± 1.22E−03	1.55E+01 ± 1.18E+01	8.96E−01 ± 3.65E−02	1.05E+02 ± 4.46E+00	**3.78E−06 ± 3.52E−06**
7	5.98E+02 ± 3.47E+01	2.00E+02 ± 1.64E+01	**1.99E+02 ± 2.83E+01**	2.30E+02 ± 2.38E+01	2.05E+03 ± 1.71E+02	3.70E+02 ± 1.41E+01
8	4.29E+02 ± 1.87E+01	1.34E+02 ± 1.35E+01	2.33E+02 ± 3.73E+01	**8.71E+01 ± 1.82E+01**	8.06E+02 ± 2.61E+01	3.24E+02 ± 1.36E+01
9	7.94E+03 ± 2.50E+03	3.94E+00 ± 4.75E+00	6.53E+03 ± 2.49E+03	5.69E+02 ± 3.49E+02	4.52E+04 ± 5.03E+03	**1.07E−02 ± 2.79E−02**
10	1.19E+04 ± 7.11E+02	6.96E+03 ± 4.28E+02	5.56E+03 ± 8.76E+02	**4.11E+03 ± 7.06E+02**	1.36E+04 ± 3.20E+02	1.30E+04 ± 4.50E+02
11	2.49E+03 ± 9.73E+02	9.62E+01 ± 1.51E+01	1.63E+02 ± 3.71E+01	3.99E+03 ± 3.26E+03	2.00E+04 ± 3.07E+03	**3.45E+01 ± 2.32E+01**
12	3.10E+09 ± 1.12E+09	4.74E+05 ± 2.23E+05	1.87E+06 ± 1.32E+06	1.10E+07 ± 4.55E+06	1.11E+11 ± 1.89E+10	**9.46E+03 ± 7.01E+03**
13	8.03E+08 ± 2.05E+08	**1.45E+03 ± 1.00E+03**	3.93E+03 ± 4.85E+03	2.12E+06 ± 7.10E+05	6.60E+10 ± 1.62E+10	1.49E+03 ± 2.16E+03
14	8.55E+05 ± 4.68E+05	3.45E+04 ± 1.68E+04	5.32E+04 ± 3.68E+04	3.70E+06 ± 2.67E+06	8.63E+07 ± 3.34E+07	**1.24E+02 ± 1.87E+01**
15	7.83E+07 ± 3.16E+07	2.24E+03 ± 1.63E+03	5.16E+03 ± 4.69E+03	1.49E+06 ± 4.43E+05	2.04E+10 ± 7.77E+09	**4.20E+01 ± 1.68E+01**
16	2.36E+03 ± 4.13E+02	**1.05E+03 ± 1.69E+02**	1.50E+03 ± 3.84E+02	1.70E+03 ± 3.68E+02	5.80E+03 ± 3.21E+02	1.83E+03 ± 6.59E+02
17	1.77E+03 ± 2.72E+02	**7.37E+02 ± 1.11E+02**	1.14E+03 ± 2.81E+02	1.20E+03 ± 3.13E+02	5.14E+03 ± 6.89E+02	1.35E+03 ± 1.90E+02
18	4.27E+06 ± 4.18E+06	6.43E+05 ± 2.78E+05	1.02E+06 ± 5.61E+05	8.01E+06 ± 5.70E+06	5.13E+08 ± 5.72E+08	**5.98E+02 ± 3.37E+02**
19	5.60E+07 ± 2.37E+07	1.01E+04 ± 3.71E+03	1.36E+04 ± 7.22E+03	6.51E+05 ± 1.96E+05	9.17E+09 ± 2.48E+09	**3.05E+01 ± 9.59E+00**
20	1.23E+03 ± 2.25E+02	**5.02E+02 ± 9.84E+01**	9.06E+02 ± 2.68E+02	1.16E+03 ± 3.44E+02	2.05E+03 ± 1.01E+02	1.37E+03 ± 1.28E+02
21	6.12E+02 ± 2.69E+01	**3.27E+02 ± 1.58E+01**	4.33E+02 ± 4.96E+01	2.96E+02 ± 1.58E+01	1.04E+03 ± 3.00E+01	5.21E+02 ± 1.31E+01
22	1.21E+04 ± 1.91E+03	5.65E+03 ± 3.18E+03	6.02E+03 ± 2.58E+03	**4.92E+03 ± 8.04E+02**	1.38E+04 ± 3.66E+02	1.10E+04 ± 4.85E+03
23	8.99E+02 ± 3.04E+01	5.60E+02 ± 1.49E+01	8.06E+02 ± 1.03E+02	5.48E+02 ± 2.41E+01	1.74E+03 ± 8.83E+01	**5.42E+02 ± 1.39E+02**
24	9.54E+02 ± 3.00E+01	**6.13E+02 ± 1.50E+01**	8.83E+02 ± 8.98E+01	5.88E+02 ± 1.77E+01	1.96E+03 ± 1.08E+02	6.34E+02 ± 1.39E+02
25	1.31E+03 ± 2.68E+02	5.62E+02 ± 2.87E+01	5.57E+02 ± 2.37E+01	5.75E+02 ± 2.68E+01	1.65E+04 ± 1.61E+03	**5.56E+02 ± 4.62E+01**
26	5.87E+03 ± 2.68E+02	2.54E+03 ± 2.28E+02	1.94E+03 ± 2.23E+03	2.30E+03 ± 1.77E+02	1.57E+04 ± 8.19E+02	**1.27E+03 ± 9.18E+01**
27	9.15E+02 ± 8.20E+01	6.00E+02 ± 2.82E+01	7.51E+02 ± 1.09E+02	7.48E+02 ± 7.01E+01	2.80E+03 ± 1.80E+02	**5.92E+02 ± 8.29E+01**
28	1.41E+03 ± 2.99E+02	5.02E+02 ± 2.01E+01	5.12E+02 ± 3.21E+01	5.26E+02 ± 1.78E+01	1.03E+04 ± 6.44E+02	**4.94E+02 ± 2.24E+01**
29	2.35E+03 ± 3.11E+02	6.58E+02 ± 1.05E+02	1.27E+03 ± 2.56E+02	1.12E+03 ± 2.31E+02	8.78E+03 ± 1.30E+03	**3.60E+02 ± 2.23E+01**
30	2.12E+08 ± 5.37E+07	8.16E+05 ± 5.64E+04	9.06E+05 ± 1.24E+05	1.83E+06 ± 3.95E+05	1.31E+10 ± 3.91E+09	**5.96E+05 ± 2.24E+04**

compared with others. Generally, GSK, AMO and BBO do significantly better than the others on most functions.

Secondly, the performance of GSK and other competitive algorithms is discussed. Table 27 clearly shows that AMO gets the first ranking among all algorithms, followed by SFS and GSK in second and third place, respectively. However, PSO, ES and ACO are the poorest algorithms, respectively. The ranking of all algorithms on the CEC 2017 functions is shown in Fig. 13. Table 28 summarizes the statistical analysis results of applying multiple-problem Wilcoxon's test between GSK and other compared algorithms on CEC2011 problems.

From Table 28, we can see that GSK obtains higher R^+ values than R^- in all the cases with exception to AMO which means that GSK is better than 9 algorithms out of 10 algorithms on 22 test functions. Precisely, we can draw the following conclusions: GSK outperforms GA, DE, GWO, PSO, ES and ACO significantly while GSK is insignificantly better than SFS, TLBO and BBO.

From the above results, comparisons and discussion through this section, the proposed GSK algorithm is of better searching quality, efficiency and robustness for solving small, moderate and high dimensions real-world unconstrained global optimization problems.

Furthermore, in order to analyze the convergence behavior of GSK and other state-of-the-art algorithms, the convergence characteristics in terms of the best fitness value of the median run of all algorithms for all functions is illustrated in the supplemental file (Fig. S2). It is clear that the convergence speed of the GSK algorithm is fast at the early stage of the optimization process for all functions with different shapes, complexity, and dimensions. Furthermore, the convergence speed is dramatically decreased, and its improvement is found to be significant in the middle and later stages of the optimization process.

Additionally, the convergent figure suggests that the GSK algorithm can reach the global solution or better solution in most problems in a fewer number of generations less than the maximum predetermined number of generations. In

Table 17 Experimental results of TLBO, SFS, DE, GA, ES and GSK over 51 independent runs on 29 test functions of 100 variables with 1,000,000 FES

Function	TLBO	SFS	DE	GA	ES	GSK
1	7.13E+03 ± 8.54E+03	8.57E+03 ± 1.21E+04	1.10E+04 ± 1.62E+04	2.03E+08 ± 1.62E+08	5.41E+11 ± 2.51E+10	**5.80E+03 ± 4.63E+03**
3	6.30E+04 ± 1.28E+04	**3.00E+04 ± 6.89E+03**	4.23E+05 ± 2.81E+04	2.07E+05 ± 3.03E+04	9.06E+05 ± 1.15E+05	1.15E+05 ± 2.15E+04
4	2.70E+02 ± 5.22E+01	2.72E+02 ± 3.84E+01	2.16E+02 ± 2.36E+01	4.76E+02 ± 6.04E+01	2.10E+05 ± 2.61E+04	**2.05E+02 ± 4.57E+01**
5	5.90E+02 ± 5.32E+01	6.36E+02 ± 9.34E+01	8.19E+02 ± 1.77E+01	1.14E+03 ± 7.63E+01	2.37E+03 ± 6.77E+01	**5.31E+02 ± 3.39E+02**
6	4.20E+01 ± 3.90E+00	1.76E+00 ± 1.94E+00	2.57E−03 ± 4.31E−03	4.24E+01 ± 4.85E+00	1.47E+02 ± 5.47E+00	**1.72E−03 ± 6.46E−03**
7	1.35E+03 ± 2.09E+02	9.78E+02 ± 1.76E+02	9.39E+02 ± 1.95E+01	1.60E+03 ± 1.19E+02	1.16E+04 ± 5.35E+02	**8.75E+02 ± 1.83E+01**
8	6.31E+02 ± 5.78E+01	6.27E+02 ± 9.44E+01	8.18E+02 ± 2.02E+01	1.08E+03 ± 9.95E+01	2.48E+03 ± 8.28E+01	**4.92E+02 ± 3.41E+02**
9	2.82E+04 ± 9.68E+03	1.18E+04 ± 3.78E+03	**3.86E+00 ± 4.56E+00**	3.11E+04 ± 7.46E+03	1.78E+05 ± 8.70E+03	8.46E+00 ± 3.41E+00
10	2.30E+04 ± 6.19E+03	**1.21E+04 ± 1.33E+03**	3.01E+04 ± 3.96E+02	2.28E+04 ± 1.51E+03	3.14E+04 ± 4.56E+02	2.95E+04 ± 4.44E+02
11	1.16E+03 ± 2.76E+02	6.43E+02 ± 8.04E+01	6.05E+02 ± 8.70E+01	6.12E+04 ± 2.52E+04	3.92E+05 ± 5.99E+04	**2.78E+02 ± 6.85E+01**
12	1.46E+06 ± 9.86E+05	2.11E+06 ± 9.27E+05	2.72E+05 ± 1.28E+05	8.67E+07 ± 3.86E+07	2.74E+11 ± 2.49E+10	**8.34E+04 ± 7.52E+04**
13	8.96E+03 ± 4.36E+03	6.42E+03 ± 6.05E+03	5.32E+03 ± 4.39E+03	2.75E+06 ± 2.82E+06	6.11E+10 ± 9.99E+09	**3.20E+03 ± 2.64E+03**
14	1.70E+05 ± 1.96E+05	**3.37E+02 ± 4.29E+01**	7.71E+03 ± 8.97E+03	9.93E+06 ± 5.46E+06	1.13E+08 ± 5.11E+07	4.64E+03 ± 4.47E+03
15	2.41E+03 ± 2.13E+03	3.12E+03 ± 3.95E+03	8.91E+03 ± 7.19E+03	1.78E+06 ± 4.52E+05	2.95E+10 ± 5.17E+09	**7.33E+02 ± 1.09E+03**
16	3.36E+03 ± 6.41E+02	3.35E+03 ± 6.00E+02	7.65E+03 ± 3.37E+02	6.22E+03 ± 7.26E+02	2.46E+04 ± 2.85E+03	**2.27E+03 ± 2.61E+03**
17	3.17E+03 ± 5.95E+02	**2.30E+03 ± 4.85E+02**	4.71E+03 ± 4.85E+02	3.70E+03 ± 4.74E+02	1.05E+07 ± 4.84E+06	3.91E+03 ± 6.68E+02
18	4.27E+05 ± 2.51E+05	**3.18E+04 ± 2.32E+04**	1.21E+05 ± 6.33E+04	8.40E+06 ± 4.08E+06	3.32E+08 ± 9.55E+07	5.73E+04 ± 3.63E+04
19	2.36E+03 ± 2.03E+03	3.39E+03 ± 4.82E+03	8.95E+03 ± 1.05E+04	1.30E+06 ± 2.09E+05	3.10E+10 ± 4.10E+09	**1.00E+03 ± 8.30E+02**
20	2.34E+03 ± 8.22E+02	**2.04E+03 ± 4.66E+02**	4.15E+03 ± 8.41E+02	3.92E+03 ± 4.42E+02	6.51E+03 ± 2.21E+02	4.46E+03 ± 2.22E+02
21	8.80E+02 ± 8.83E+01	7.12E+02 ± 7.84E+01	1.04E+03 ± 2.54E+01	1.48E+03 ± 1.20E+02	2.90E+03 ± 1.12E+02	**6.07E+02 ± 3.37E+02**
22	2.49E+04 ± 8.31E+03	**1.30E+04 ± 5.94E+03**	3.02E+04 ± 7.15E+02	2.41E+04 ± 1.07E+03	3.19E+04 ± 4.29E+02	3.00E+04 ± 4.57E+02
23	1.35E+03 ± 1.05E+02	9.75E+02 ± 7.94E+01	8.60E+02 ± 2.99E+02	1.54E+03 ± 1.02E+02	3.54E+03 ± 8.56E+01	**6.11E+02 ± 1.58E+01**
24	1.99E+03 ± 1.96E+02	1.54E+03 ± 1.00E+02	1.63E+03 ± 1.49E+02	2.11E+03 ± 1.33E+02	5.46E+03 ± 1.17E+02	**9.32E+02 ± 1.70E+01**
25	8.28E+02 ± 6.11E+01	7.90E+02 ± 4.99E+01	**7.31E+02 ± 5.56E+01**	1.63E+03 ± 1.63E+02	1.04E+05 ± 1.09E+04	8.21E+02 ± 4.34E+01
26	2.01E+04 ± 5.23E+03	1.37E+04 ± 6.56E+03	1.08E+04 ± 1.37E+03	1.58E+04 ± 1.46E+03	5.10E+04 ± 1.66E+03	**3.66E+03 ± 1.78E+02**
27	1.42E+03 ± 2.02E+02	8.81E+02 ± 7.99E+01	**6.16E+02 ± 2.15E+01**	1.27E+03 ± 1.20E+02	6.72E+03 ± 6.63E+02	6.57E+02 ± 3.07E+01
28	6.30E+02 ± 2.91E+01	6.13E+02 ± 3.09E+01	5.59E+02 ± 3.52E+01	2.60E+03 ± 1.62E+03	4.88E+04 ± 2.86E+03	**5.53E+02 ± 3.25E+01**
29	4.34E+03 ± 6.73E+02	3.09E+03 ± 4.26E+02	4.74E+03 ± 1.19E+03	4.37E+03 ± 4.85E+02	2.49E+06 ± 1.36E+06	**1.21E+03 ± 1.74E+02**
30	2.15E+04 ± 2.11E+04	1.22E+04 ± 6.93E+03	4.68E+03 ± 3.05E+03	3.17E+06 ± 3.02E+05	4.85E+10 ± 9.66E+09	**2.99E+03 ± 2.73E+02**

general, GSK is scalable enough and can balance greatly the exploration and exploitation abilities until the maximum FEs is reached. Therefore, the proposed GSK algorithm is proven to be an effective and powerful approach for solving unconstrained global optimization problems within limited number of function evaluations which is a very important issue when dealing with real-world problems.

4 Conclusion

The gaining and sharing knowledge (GSK) algorithm is proposed as a novel metaheuristic for solving optimization problems. The GSK is a population based stochastic algorithm that mimics the process of gaining and sharing knowledge during the human life span. It is based on two vital stages, junior gaining-sharing phase and senior gaining-sharing phase. In the proposed GSK algorithm, each candidate solution is presented as a person in the entire population of people. This person has different level of knowledge in various disciplines and fields by utilization of both junior and senior phases, where each discipline represents a specific dimension of the optimized problem.

During Junior phase, the person gains and shares knowledge from/with small private and social networks while he cooperates and competes with different types of larger networks of people with various talents, experience, characteristics in senior phase which are the source of gaining and sharing knowledge.

The mathematical expressions of this process is formulated. In order to test the effectiveness of GSK, it is applied to solve the CEC-2017 real-parameter benchmark optimization problems. Experimental results are compared with state-of-the-art algorithms which were 4 evolutionary algorithms like SFS, DE, GA and ES and 1 human related algorithm

Table 18 Experimental results of GWO, AMO, PSO, BBO, ACO and GSK over 51 independent runs on 29 test functions of 100 variables with 1,000,000 FES

Function	GWO	AMO	PSO	BBO	ACO	GSK
1	5.72E+10 ± 6.63E+09	**2.51E+03 ± 1.89E+03**	1.08E+04 ± 1.42E+04	1.28E+07 ± 5.83E+05	4.99E+11 ± 1.38E+11	5.80E+03 ± 4.63E+03
3	2.08E+05 ± 1.74E+04	1.75E+05 ± 1.41E+04	**3.17E+04 ± 5.19E+03**	4.21E+05 ± 8.31E+04	1.32E+11 ± 2.81E+11	**1.15E+05 ± 2.15E+04**
4	4.68E+03 ± 8.31E+02	**1.41E+02 ± 6.04E+01**	3.03E+02 ± 3.91E+01	2.87E+02 ± 4.43E+01	1.28E+05 ± 1.09E+04	2.05E+02 ± 4.57E+01
5	1.09E+03 ± 4.01E+01	4.41E+02 ± 4.10E+01	6.02E+02 ± 6.13E+01	**2.24E+02 ± 3.29E+01**	1.98E+03 ± 3.89E+01	5.31E+02 ± 3.39E+02
6	5.82E+01 ± 3.57E+00	6.28E−02 ± 8.93E−02	3.64E+01 ± 8.16E+00	8.79E−01 ± 4.51E−02	1.35E+02 ± 3.70E+00	**1.72E−03 ± 6.46E−03**
7	1.69E+03 ± 7.81E+01	5.86E+02 ± 4.46E+01	**5.71E+02 ± 1.17E+02**	6.31E+02 ± 5.66E+01	8.86E+03 ± 1.95E+02	8.75E+02 ± 1.83E+01
8	1.08E+03 ± 3.93E+01	4.33E+02 ± 4.24E+01	6.01E+02 ± 7.76E+01	**2.28E+02 ± 3.48E+01**	2.00E+03 ± 2.18E+01	4.92E+02 ± 3.41E+02
9	3.79E+04 ± 4.59E+03	1.83E+03 ± 1.30E+03	1.81E+04 ± 3.09E+03	2.25E+03 ± 9.01E+02	1.25E+05 ± 7.21E+03	**8.46E+00 ± 3.41E+00**
10	2.84E+04 ± 8.64E+02	1.81E+04 ± 7.17E+02	1.32E+04 ± 1.28E+03	**1.10E+04 ± 9.31E+02**	3.01E+04 ± 7.44E+02	2.95E+04 ± 4.44E+02
11	4.19E+04 ± 9.04E+03	6.14E+02 ± 7.28E+01	1.12E+03 ± 2.00E+02	6.48E+04 ± 1.92E+04	1.17E+07 ± 3.79E+07	**2.78E+02 ± 6.85E+01**
12	1.55E+10 ± 2.53E+09	1.34E+06 ± 5.48E+05	1.11E+07 ± 5.28E+06	4.10E+07 ± 1.51E+07	3.32E+11 ± 3.41E+10	**8.34E+04 ± 7.52E+04**
13	2.43E+09 ± 6.66E+08	**2.37E+03 ± 1.03E+03**	4.22E+03 ± 3.95E+03	2.20E+06 ± 3.17E+05	8.01E+10 ± 1.35E+10	3.20E+03 ± 2.64E+03
14	6.79E+06 ± 2.94E+06	8.50E+05 ± 3.54E+05	5.24E+05 ± 2.42E+05	1.54E+07 ± 7.21E+06	2.74E+08 ± 2.30E+08	**4.64E+03 ± 4.47E+03**
15	7.11E+08 ± 1.87E+08	**5.84E+02 ± 3.50E+02**	2.17E+03 ± 1.59E+03	1.37E+06 ± 2.99E+05	3.65E+10 ± 5.65E+09	7.33E+02 ± 1.09E+03
16	7.91E+03 ± 7.06E+02	3.28E+03 ± 2.77E+02	3.31E+03 ± 5.55E+02	3.44E+03 ± 7.75E+02	1.88E+04 ± 1.35E+03	**2.27E+03 ± 2.61E+03**
17	6.14E+03 ± 4.78E+02	**2.40E+03 ± 2.29E+02**	2.93E+03 ± 5.54E+02	3.20E+03 ± 7.82E+02	4.34E+06 ± 2.67E+06	3.91E+03 ± 6.68E+02
18	9.63E+06 ± 3.30E+06	1.60E+06 ± 4.67E+05	1.92E+06 ± 8.57E+05	6.60E+06 ± 3.01E+06	5.51E+08 ± 1.57E+08	**5.73E+04 ± 3.63E+04**
19	6.23E+08 ± 1.19E+08	1.24E+03 ± 9.31E+02	1.93E+03 ± 2.77E+03	1.39E+06 ± 2.01E+05	3.35E+10 ± 8.97E+09	**1.00E+03 ± 8.30E+02**
20	4.30E+03 ± 4.60E+02	**2.30E+03 ± 2.63E+02**	2.75E+03 ± 4.15E+02	2.74E+03 ± 4.91E+02	5.42E+03 ± 1.66E+02	4.46E+03 ± 2.22E+02
21	1.30E+03 ± 5.09E+01	6.23E+02 ± 2.31E+01	9.70E+02 ± 1.34E+02	**4.78E+02 ± 3.30E+01**	2.62E+03 ± 6.65E+01	6.07E+02 ± 3.37E+02
22	2.95E+04 ± 9.92E+02	1.94E+04 ± 7.43E+02	1.55E+04 ± 1.75E+03	**1.24E+04 ± 1.12E+03**	3.19E+04 ± 5.87E+02	3.00E+04 ± 4.57E+02
23	1.66E+03 ± 5.24E+01	8.22E+02 ± 2.46E+01	1.49E+03 ± 1.49E+02	7.11E+02 ± 3.06E+01	3.91E+03 ± 1.67E+02	**6.11E+02 ± 1.58E+01**
24	2.19E+03 ± 8.62E+01	1.23E+03 ± 4.73E+01	1.60E+03 ± 1.56E+02	1.20E+03 ± 4.17E+01	7.41E+03 ± 6.15E+02	**9.32E+02 ± 1.70E+01**
25	3.84E+03 ± 5.24E+02	8.31E+02 ± 5.21E+01	8.12E+02 ± 7.29E+01	**8.03E+02 ± 7.33E+01**	4.90E+04 ± 5.90E+03	8.21E+02 ± 4.34E+01
26	1.65E+04 ± 6.27E+02	7.91E+03 ± 6.30E+02	9.73E+03 ± 5.69E+03	6.26E+03 ± 4.03E+02	5.23E+04 ± 2.10E+03	**3.66E+03 ± 1.78E+02**
27	1.60E+03 ± 1.20E+02	8.49E+02 ± 5.06E+01	9.31E+02 ± 1.14E+02	8.28E+02 ± 6.41E+01	7.92E+03 ± 2.58E+02	**6.57E+02 ± 3.07E+01**
28	5.23E+03 ± 9.72E+02	5.60E+02 ± 3.26E+01	6.55E+02 ± 4.30E+01	6.44E+02 ± 4.08E+01	3.82E+04 ± 1.51E+03	**5.53E+02 ± 3.25E+01**
29	7.49E+03 ± 5.12E+02	2.74E+03 ± 3.04E+02	3.66E+03 ± 5.24E+02	3.29E+03 ± 5.09E+02	1.19E+05 ± 5.53E+04	**1.21E+03 ± 1.74E+02**
30	1.95E+09 ± 4.30E+08	5.25E+03 ± 1.28E+03	8.19E+03 ± 3.68E+03	2.63E+06 ± 4.88E+05	6.75E+10 ± 1.47E+10	**2.99E+03 ± 2.73E+02**

Table 19 Ranking of algorithms using score metric on the CEC 2017 functions

Algorithm	Score1	Score2	Score	Ranking
GSK	50	47.16667	97.17	1
DE	27.10	33.27	60.37	2
AMO	7.49	50	57.49	3
SFS	12.88	40.01	52.89	4
TLBO	10.38	28.22	38.60	5
PSO	2.53	29.44	31.97	6
BBO	0.36	26.90	27.26	7
GA	0.11	18.06	18.17	8
GWO	4.58E−04	17.01	17.01	9
ACO	2.73E−05	13.75	13.76	10
ES	3.25E−05	13.42	13.42	11

TLBO and 5 swarm intelligence algorithms like GWO, ACO, BBO, AMO and PSO. In order to evaluate the performance of each algorithm, a score metric which is recently defined for the CEC 2017 competition is used. It takes into account the error values for all dimensions and the rank for each problem in each dimension.

GSK gets the first ranking among all algorithms, followed by DE and AMO in second and third place, respectively. Furthermore, in order to statistically analyze the performance of GSK, non-parametric tests (the Wilcoxon's test) are used with the significance level of 0.05.

As a summary of results, the performance of the GSK algorithm was statistically superior to and competitive with other recent and well-known state-of-the-art algorithms in the majority of functions and for different dimensions especially in high dimensions.

Table 20 Results of multiple-problem Wilcoxon's test between GSK and other algorithms for D = 10, 30,50 and 100

D	Algorithms	R^+	R^-	p value	+	≈	−	Dec.
10	GSK vs TLBO	298.5	107.5	**0.030**	19	1	9	+
	GSK vs SFS	245	190	0.552	17	0	12	≈
	GSK vs DE	81	172	0.140	7	7	15	≈
	GSK vs GA	403	32	**0.000**	27	0	2	+
	GSK vs ES	405	1	**0.000**	27	1	1	+
	GSK vs GWO	416	19	**0.000**	28	0	1	+
	GSK vs AMO	155	196	0.603	12	3	14	≈
	GSK vs PSO	294.5	83.5	**0.011**	18	2	9	+
	GSK vs BBO	368.5	66.5	**0.001**	23	0	6	+
	GSK vs ACO	424.5	10.5	**0.000**	27	0	2	+
30	GSK vs TLBO	340	95	**0.005**	21	0	8	+
	GSK vs SFS	237	169	0.439	17	1	11	≈
	GSK vs DE	238	113	0.112	17	3	9	≈
	GSK vs GA	417	18	**0.000**	28	0	1	+
	GSK vs ES	406	0	**0.000**	28	1	0	+
	GSK vs GWO	420	15	**0.000**	28	0	1	+
	GSK vs AMO	228	150	0.350	14	2	13	≈
	GSK vs PSO	382	53	**0.000**	24	0	5	+
	GSK vs BBO	366	69	**0.001**	22	0	7	+
	GSK vs ACO	435	0	**0.000**	29	0	0	+
50	GSK vs TLBO	310	125	**0.045**	20	0	9	+
	GSK vs SFS	251	184	0.469	19	0	10	≈
	GSK vs DE	268	138	0.139	18	1	10	≈
	GSK vs GA	400	35	**0.000**	26	0	3	+
	GSK vs ES	435	0	**0.000**	29	0	0	+
	GSK vs GWO	414.5	20.5	**0.000**	27	0	2	+
	GSK vs AMO	264	171	0.315	17	0	12	≈
	GSK vs PSO	304	131	0.061	19	0	10	≈
	GSK vs BBO	317	118	**0.031**	19	0	10	+
	GSK vs ACO	435	0	**0.000**	29	0	0	+
100	GSK vs TLBO	339	96	**0.009**	24	0	5	+
	GSK vs SFS	274	161	0.222	21	0	8	≈
	GSK vs DE	410	25	**0.000**	25	0	4	+
	GSK vs GA	396	39	**0.000**	25	0	4	+
	GSK vs ES	435	0	**0.000**	29	0	0	+
	GSK vs GWO	420	15	**0.000**	26	0	3	+
	GSK vs AMO	274	161	0.222	18	0	11	≈
	GSK vs PSO	321	114	**0.025**	22	0	7	+
	GSK vs BBO	336	99	**0.010**	20	0	9	+
	GSK vs ACO	435	0	**0.000**	29	0	0	+

Besides, the GSK algorithm has been applied to solve the set of real world optimization problems proposed for the IEEE-CEC2011 evolutionary algorithm competition. Generally, GSK, AMO and BBO do significantly better than the others on most functions. Virtually, it is easily implemented and has been proven to be a reliable approach for real parameter optimization.

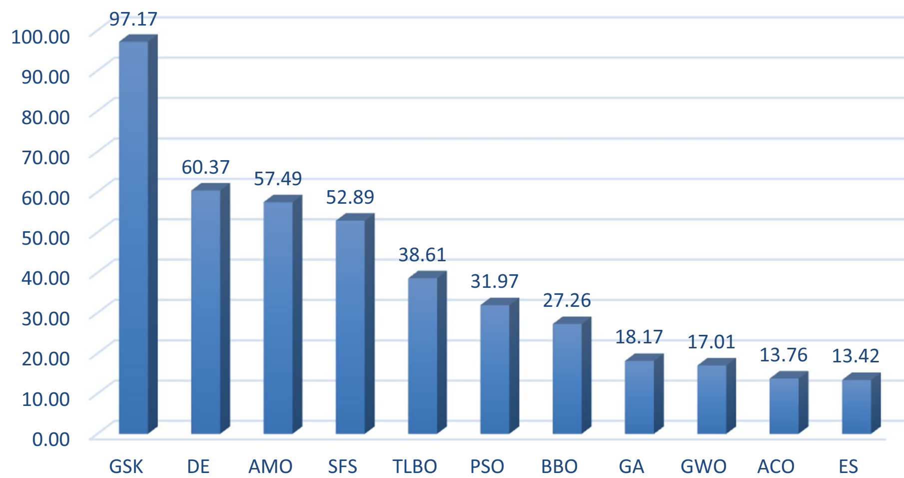

Fig. 11 The score of all algorithms on the CEC 2017 functions

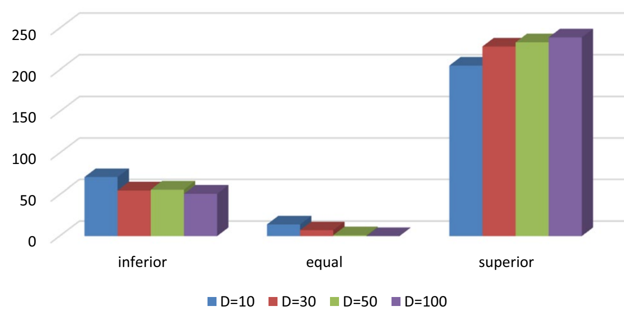

Fig. 12 Statistical comparison results of GSK against other state-of-the-art algorithms with the growth of the dimensionality

Table 21 Algorithm complexity results for D = 10

Alg.	T_0	T_1	T_2	$(T_2-T_1)/T_0$
GSK	0.0398	0.5934	1.916	33.2311558
PSO			9.7927	231.138191
TLBO			4.85	106.949749
GWO			2.787	55.1155779
SFS			4.6507	101.942211
AMO			7.2405	167.012563
ES			8.2422	192.180905
DE			4.0051	85.721105
GA			54.1597	1345.88693
ACO			23.3552	571.904523
BBO			26.9809	663.002513

Table 22 Algorithm complexity results for D = 30

Alg.	T_0	T_1	T_2	$(T_2-T_1)/T_0$
GSK	0.0398	0.7577	2.3803	40.7674945
PSO			9.969	231.438349
TLBO			5.159	110.584077
GWO			3.3747	65.7524191
SFS			5.0192	107.071515
AMO			17.1161	411.013726
ES			16.7547	401.933324
DE			6.8723	113.652632
GA			51.9629	1286.56146
ACO			67.0559	1665.78257
BBO			36.5998	900.553927

Table 23 Algorithm complexity results for D = 50

Alg.	T_0	T_1	T_2	$(T_2-T_1)/T_0$
GSK	0.0398	1.0077	3.0954	52.4547739
PSO			10.3271	234.155779
TLBO			5.5209	113.396985
GWO			4.2553	81.5979899
SFS			5.6214	115.922111
AMO			26.8281	648.753769
ES			27.0341	653.929648
DE			9.9907	225.703517
GA			51.8108	1276.4598
ACO			111.4702	2775.4397
BBO			46.9975	1155.52261

Table 24 Result of GSK on CEC2011

Function	Best	Median	Mean	Worst	SD
1	0.00E+00	1.85E−21	3.28E+00	1.49E+01	5.21E+00
2	−1.35E+01	−1.14E+01	−1.13E+01	−9.25E+00	1.03E+00
3	1.15E−05	1.15E−05	1.15E−05	1.15E−05	9.12E−13
4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
5	−2.39E+01	−2.08E+01	−2.06E+01	−1.87E+01	1.21E+00
6	−1.16E+01	−6.85E+00	−6.94E+00	0.00E+00	2.48E+00
7	1.59E+00	1.78E+00	1.78E+00	1.97E+00	1.08E−01
8	2.20E+02	2.20E+02	2.20E+02	2.20E+02	0.00E+00
9	1.29E+03	2.07E+03	2.11E+03	3.09E+03	5.02E+02
10	−2.18E+01	−2.16E+01	−2.16E+01	−2.14E+01	1.19E−01
11	5.09E+04	5.24E+04	5.24E+04	5.39E+04	6.88E+02
12	1.07E+06	1.07E+06	1.07E+06	1.08E+06	1.73E+03
13	1.54E+04	1.54E+04	1.54E+04	1.55E+04	2.44E+00
14	1.82E+04	1.84E+04	1.84E+04	1.86E+04	1.22E+02
15	3.28E+04	3.28E+04	3.28E+04	3.28E+04	1.55E+01
16	1.32E+05	1.35E+05	1.35E+05	1.39E+05	2.22E+03
17	1.97E+06	2.03E+06	2.09E+06	2.36E+06	1.20E+05
18	1.17E+06	1.27E+06	1.27E+06	1.57E+06	7.56E+04
19	1.75E+06	2.03E+06	2.00E+06	2.25E+06	1.36E+05
20	1.12E+06	1.29E+06	1.29E+06	1.46E+06	9.20E+04
21	1.34E+01	1.61E+01	1.70E+01	2.49E+01	3.11E+00
22	8.61E+00	1.28E+01	1.29E+01	2.09E+01	2.93E+00

Table 25 Experimental results of TLBO, SFS, DE, GA, ES and GSK over 25 independent runs on 22 test functions with 150,000 FES

Func.	TLBO	SFS	DE	GA	ES	GSK
1	6.60E+00 ± 6.78E+00	4.69E+00 ± 4.49E+00	4.30E+00 ± 5.38E+00	1.95E+01 ± 3.01E+00	2.84E+01 ± 1.31E+00	**3.28E+00 ± 5.21E+00**
2	−2.08E+01 ± 2.15E+00	**−2.66E+01 ± 1.28E+00**	−1.31E+01 ± 4.60E+00	−9.87E+00 ± 2.20E+00	−2.54E+00 ± 3.38E−01	−1.13E+01 ± 1.03E+00
3	**1.15E−05 ± 4.08E−17**	**1.15E−05 ± 4.87E−10**	**1.15E−05 ± 1.86E−15**	**1.15E−05 ± 9.88E−10**	**1.15E−05 ± 0.00E+00**	**1.15E−05 ± 9.12E−13**
4	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**
5	−2.83E+01 ± 3.70E+00	**−3.37E+01 ± 1.61E+00**	−1.95E+01 ± 9.34E−01	−1.38E+01 ± 1.94E+00	−2.76E+01 ± 3.23E+00	−2.06E+01 ± 1.21E+00
6	−1.94E+01 ± 1.70E+00	**−2.74E+01 ± 1.86E+00**	−1.41E+01 ± 1.53E+00	−3.35E+00 ± 3.99E+00	−1.99E+01 ± 6.06E+00	−6.94E+00 ± 2.48E+00
7	1.16E+00 ± 2.69E−01	1.36E+00 ± 1.39E−01	1.75E+00 ± 9.79E−02	**1.13E+00 ± 1.61E−01**	2.44E+00 ± 1.98E−01	1.78E+00 ± 1.08E−01
8	**2.20E+02 ± 0.00E+00**	**2.20E+02 ± 0.00E+00**	**2.20E+02 ± 0.00E+00**	**2.20E+02 ± 0.00E+00**	5.55E+02 ± 3.15E+02	**2.20E+02 ± 0.00E+00**
9	3.99E+03 ± 2.88E+03	2.27E+04 ± 6.55E+03	2.37E+04 ± 2.35E+03	1.32E+05 ± 3.76E+04	2.00E+06 ± 6.38E+04	**2.11E+03 ± 5.02E+02**
10	−1.89E+01 ± 2.43E+00	−2.15E+01 ± 1.31E−01	−1.41E+01 ± 2.68E+00	−9.61E+00 ± 1.67E+00	−8.90E+00 ± 3.55E−01	**−2.16E+01 ± 1.19E−01**
11	3.79E+06 ± 9.41E+05	4.92E+05 ± 2.14E+05	1.26E+05 ± 2.43E+04	5.52E+05 ± 8.33E+05	2.91E+08 ± 2.42E+07	**5.24E+04 ± 6.88E+02**
12	1.53E+06 ± 2.23E+05	1.30E+06 ± 6.45E+04	1.14E+06 ± 9.67E+03	1.05E+07 ± 7.73E+05	1.45E+07 ± 6.02E+05	**1.07E+06 ± 1.73E+03**
13	1.55E+04 ± 1.24E+01	**1.54E+04 ± 1.44E+00**	**1.54E+04 ± 1.18E+01**	1.55E+04 ± 2.23E+01	1.81E+04 ± 4.44E+03	**1.54E+04 ± 2.44E+00**
14	1.93E+04 ± 9.32E+01	1.88E+04 ± 8.04E+01	**1.84E+04 ± 1.66E+02**	1.98E+04 ± 7.01E+02	1.88E+04 ± 3.71E−12	**1.84E+04 ± 1.22E+02**
15	3.29E+04 ± 7.14E+01	3.30E+04 ± 2.35E+01	3.29E+04 ± 3.03E+01	3.30E+04 ± 7.93E+01	2.23E+05 ± 8.85E+04	**3.28E+04 ± 1.55E+01**
16	1.36E+05 ± 3.40E+03	1.37E+05 ± 2.16E+03	1.37E+05 ± 2.91E+03	1.50E+05 ± 7.18E+03	1.46E+05 ± 5.07E+03	**1.35E+05 ± 2.22E+03**
17	**2.05E+06 ± 2.09E+05**	2.21E+06 ± 2.60E+05	2.26E+06 ± 2.34E+05	8.78E+08 ± 1.10E+09	9.93E+09 ± 1.45E+09	2.09E+06 ± 1.20E+05
18	1.14E+06 ± 8.67E+04	**1.05E+06 ± 4.38E+04**	1.88E+06 ± 3.06E+05	1.32E+06 ± 2.70E+05	5.04E+07 ± 6.79E+06	1.27E+06 ± 7.56E+04
19	**1.42E+06 ± 1.58E+05**	1.51E+06 ± 1.93E+05	2.52E+06 ± 2.58E+05	2.24E+06 ± 2.48E+06	4.93E+07 ± 2.98E+06	2.00E+06 ± 1.36E+05
20	1.12E+06 ± 8.06E+04	**1.05E+06 ± 4.64E+04**	1.77E+06 ± 2.78E+05	1.37E+06 ± 1.09E+06	4.82E+07 ± 6.01E+06	1.29E+06 ± 9.20E+04
21	**1.55E+01 ± 2.02E+00**	1.77E+01 ± 3.13E+00	1.77E+01 ± 3.62E+00	3.15E+01 ± 6.07E+00	8.75E+01 ± 1.54E+01	1.70E+01 ± 3.11E+00
22	2.12E+01 ± 2.41E+00	2.00E+01 ± 3.09E+00	1.32E+01 ± 2.78E+00	3.50E+01 ± 8.81E+00	5.99E+01 ± 6.17E+00	**1.29E+01 ± 2.93E+00**

Several current and future works can be developed from this study. Firstly, Current research efforts focus on how to modify the GSK algorithm for handling constrained and multi-objective optimization problems as well as solving

Table 26 Experimental results of GWO, AMO, PSO, BBO, ACO and GSK over 25 independent runs on 22 test functions with 150,000 FES

Func.	GWO	AMO	PSO	BBO	ACO	GSK
1	1.54E+01 ± 4.91E+00	**4.85E−01 ± 1.17E+00**	2.63E+01 ± 0.00E+00	2.05E+01 ± 3.40E+00	3.05E+01 ± 8.23E−01	<u>3.28E+00 ± 5.21E+00</u>
2	− 1.48E+01 ± 1.10E+00	**− 1.98E+01 ± 1.03E+00**	− 3.79E+00 ± 5.52E−02	− 1.85E+01 ± 1.85E+00	− 6.16E+00 ± 8.28E−01	− 1.13E+01 ± 1.03E+00
3	**1.15E−05 ± 4.61E−10**	**1.15E−05 ± 7.25E−14**	1.15E−05 ± 1.46E−12	**1.15E−05 ± 3.80E−09**	**1.15E−05 ± 0.00E+00**	**1.15E−05 ± 9.12E−13**
4	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**	**0.00E+00 ± 0.00E+00**
5	− 2.56E+01 ± 2.56E+00	**− 3.32E+01 ± 7.33E−01**	− 1.98E+01 ± 9.91E−01	<u>− 3.30E+01 ± 7.34E−01</u>	− 1.48E+01 ± 1.21E+00	− 2.06E+01 ± 1.21E+00
6	− 1.86E+01 ± 1.63E+00	<u>− 2.70E+01 ± 9.45E−01</u>	− 1.06E+01 ± 1.03E+00	**− 2.79E+01 ± 7.95E−01**	− 1.04E+01 ± 1.42E+00	− 6.94E+00 ± 2.48E+00
7	1.72E+00 ± 1.15E−01	**1.37E+00 ± 9.38E−02**	1.53E+00 ± 1.21E−01	1.44E+00 ± 8.64E−02	<u>1.89E+00 ± 9.89E−02</u>	1.78E+00 ± 1.08E−01
8	**2.33E+02 ± 1.17E+01**	**2.20E+02 ± 0.00E+00**	2.21E+02 ± 2.84E+00	**2.20E+02 ± 0.00E+00**	2.39E+03 ± 1.32E+03	**2.20E+02 ± 0.00E+00**
9	1.09E+05 ± 1.45E+04	**1.15E+03 ± 3.24E+02**	1.85E+06 ± 1.54E+05	7.83E+04 ± 2.42E+04	1.03E+06 ± 2.42E+04	<u>2.11E+03 ± 5.02E+02</u>
10	− 1.45E+01 ± 2.50E+00	<u>− 2.13E+01 ± 9.12E−02</u>	− 9.31E+00 ± 8.20E−01	− 1.52E+01 ± 1.78E+00	− 9.04E+00 ± 1.49E−01	**− 2.16E+01 ± 1.19E−01**
11	4.71E+07 ± 5.09E+07	<u>5.26E+04 ± 5.02E+02</u>	5.36E+06 ± 3.85E+05	5.66E+04 ± 8.38E+02	9.25E+06 ± 2.73E+06	**5.24E+04 ± 6.88E+02**
12	1.29E+07 ± 7.39E+05	**1.07E+06 ± 1.27E+03**	1.42E+07 ± 9.63E+05	<u>1.09E+06 ± 1.93E+04</u>	8.10E+06 ± 2.33E+05	**1.07E+06 ± 1.73E+03**
13	1.55E+04 ± 2.10E+01	**1.54E+04 ± 2.59E+00**	1.56E+04 ± 5.13E+01	1.55E+04 ± 2.49E+01	1.29E+05 ± 9.70E+04	**1.54E+04 ± 2.44E+00**
14	1.93E+04 ± 2.34E+02	<u>1.92E+04 ± 1.48E+02</u>	1.97E+04 ± 2.14E+02	1.94E+04 ± 3.21E+02	4.15E+05 ± 4.79E+05	**1.84E+04 ± 1.22E+02**
15	3.32E+04 ± 1.70E+02	<u>3.30E+04 ± 2.09E+01</u>	1.26E+05 ± 5.23E+04	3.31E+04 ± 6.90E+01	4.13E+06 ± 3.30E+06	**3.28E+04 ± 1.55E+01**
16	1.43E+05 ± 6.64E+03	<u>1.37E+05 ± 1.70E+03</u>	4.76E+06 ± 4.24E+06	1.42E+05 ± 4.91E+03	7.43E+07 ± 1.77E+07	**1.35E+05 ± 2.22E+03**
17	6.33E+09 ± 1.12E+09	**2.02E+06 ± 1.63E+05**	1.24E+10 ± 2.39E+09	2.70E+06 ± 1.98E+06	1.78E+10 ± 3.18E+09	<u>2.09E+06 ± 1.20E+05</u>
18	5.95E+06 ± 1.51E+06	<u>1.04E+06 ± 7.56E+04</u>	1.29E+08 ± 1.15E+07	**9.70E+05 ± 1.47E+04**	1.53E+08 ± 1.89E+07	1.27E+06 ± 7.56E+04
19	6.84E+06 ± 1.82E+06	<u>1.56E+06 ± 1.29E+05</u>	1.34E+08 ± 1.95E+07	**1.53E+06 ± 2.56E+05**	1.47E+08 ± 2.71E+07	2.00E+06 ± 1.36E+05
20	6.03E+06 ± 1.58E+06	<u>1.03E+06 ± 5.94E+04</u>	1.37E+08 ± 1.98E+07	**9.75E+05 ± 1.54E+04**	1.59E+08 ± 1.44E+07	1.29E+06 ± 9.20E+04
21	3.59E+01 ± 3.52E+00	<u>1.85E+01 ± 1.39E+00</u>	6.11E+01 ± 5.34E+00	2.24E+01 ± 3.32E+00	8.61E+01 ± 2.33E+01	**1.70E+01 ± 3.11E+00**
22	3.10E+01 ± 3.78E+00	2.22E+01 ± 1.81E+00	5.09E+01 ± 4.65E+00	2.52E+01 ± 2.77E+00	8.85E+01 ± 1.26E+01	**1.29E+01 ± 2.93E+00**

Table 27 Average ranks for all algorithms across all problems using CEC2011

Algorithm	Mean Rank	Ranking
AMO	3.27	1
SFS	3.48	2
GSK	3.80	3
TLBO	3.98	4
BBO	4.75	5
DE	5.05	6
GA	7.02	7
GWO	7.18	8
PSO	8.77	9
ES	8.89	10
ACO	9.82	11

practical engineering optimization problems and real-world applications.

Secondly, concerning the improvement of GSK, it would be very interesting to propose another adaptive GSK version such that each individual has its parameter values which changed adaptively during generations. Future research studies may focus on applying the algorithm to solve high dimensions or large-scale global optimization problems. Another possible direction is developing binary, discrete versions of GSK to solve mixed integer optimization problems. Finally, hybridizing the GSK algorithm with other powerful metaheuristics are thought to be promising direction. Furthermore, it will be greatly beneficial as a future direction to investigate a complete parameter tune-free adaptive GSK by combining novel adaptive

Table 28 Results of multiple-problem Wilcoxon's test between GSK and other algorithms using CEC2011

Algorithms	R^+	R^-	p value	+	≈	−	Dec.
GSK vs. TLBO	104	86	0.717	10	3	9	≈
GSK vs. SFS	100	71	0.528	11	4	7	≈
GSK vs. DE	139	14	**0.000**	14	5	3	+
GSK vs. GA	189	1	**0.000**	18	3	1	+
GSK vs. ES	203	7	**0.000**	18	2	2	+
GSK vs. GWO	199	11	**0.000**	16	2	4	+
GSK vs. AMO	53	100	0.266	7	5	10	≈
GSK vs. PSO	205	5	**0.000**	18	2	2	+
GSK vs. BBO	120	70	0.314	12	3	7	≈
GSK vs. ACO	208	2	**0.000**	19	2	1	+

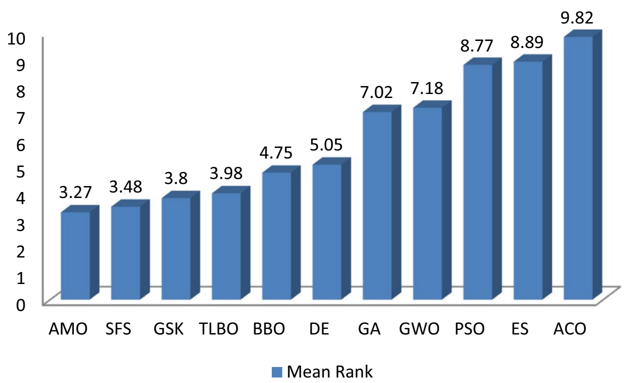

Fig. 13 The ranking of all algorithms on the CEC 2011 functions

population reduction and increment method. The Matlab source code of the proposed GSK algorithm can be downloaded from https://sites.google.com/view/optimization-project/filesoptimization-project/files.

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